A1. Write the output of the following C program and justify your answer.

```
#include<stdio.h>
void function(int a[ ]) {
  int i;
  for(i=0;i<5;i++)
      a[i]=a[i]+1;
  a=a+2;
  printf("\n%d",a[-1]);
}
int main() {
   int a[5]={5,-17,1,10,-15};
   function(a);
   printf("\n%d",a[1]);
}
```

A2. Consider the following definition of a finite set that can hold a maximum of 200 integers.

```
#define MAXSIZE 200
typedef struct {
 int data[MAXSIZE];
 int noOfElements;
} SET;
```

Let there be n elements stored in the array `data`$[0, \cdots, n-1]$ in an unordered way. The variable `noOfElements` stores the value of n. Write the following functions for the `SET` data type in C language. Invoke these functions from a main function.

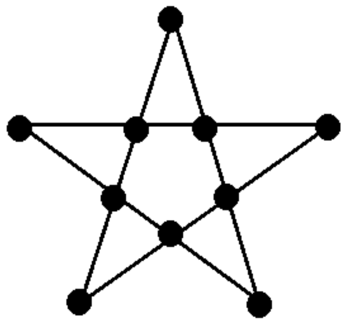
```
void initEmptySet (SET *sp);
// Initialize sp to represent an empty set

int isEmptySet (SET *sp);
// Returns 1 if sp denotes an empty set

int isAnElement (SET *sp, int x);
// Returns 1 if x is in the set denoted by sp, and
// return 0 otherwise
```

A3. Call a positive integer *good* if it is composite but not divisible by 2, 3 and 5. The three smallest good numbers are 49, 77 and 91. There are 168 prime numbers less than 1000. How many good numbers are present which are less than 1000? Justify your answer.

A4. Let S be any subset of 2023 distinct numbers chosen from the set $\{1, 2, 3, \ldots, 4044\}$. Show that S always contains a pair (a, b) such that a divides b.

A5. In a recent medical test to check for ability to distinguish between blue and green objects, Mr. Wilson was correct for 99% of the blue objects and made mistakes for 2% of the green objects. Dennis challenged Mr. Wilson to identify the lone blue ball among 99 other green balls. Mr. Wilson picked one at random and after some inspection declared it to be blue. What is the probability of Mr. Wilson being right?

A6. Consider 10 black balls arranged as shown in the figure below. Of these 10 balls, three distinct balls are chosen at random. Compute the probability that no two of the chosen balls lie on any straight line drawn in the figure.



A7. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function satisfying $f(x) = f(x^2)$ for all $x \in \mathbb{R}$. Show that f is a constant function.

A8. A function $\psi : (-1, 1) \to \mathbb{R}$ is defined as $\psi(x) = \displaystyle\int_{x^3}^{x^2} \frac{1}{1+t^4} dt$. Calculate the value of

$$\lim_{x \to 0} \frac{\psi(x)}{e^{3x^2} - 1}$$

A9. Let $n > 1$. Show that an $n \times n$ square matrix with all diagonal entries 0 and all other entries 1, is an invertible matrix.

A10. Let A be a real matrix and A^T denote its transpose. Prove that $A^T A = AA^T$ if and only if $\forall x \in \mathbb{R}^n$, $||Ax|| = ||A^T x||$, where $|| \cdot ||$ denotes the Euclidean norm.