

Model Question Paper

Junior Research Fellowship (JRF) in Computer Science (CS)

Test Code : CSA (Forenoon Examination)

Maximum marks: 60 Time: TWO hours

8th May, 2022

Answer ALL the questions.

A1. Let there be a pile of 2022 chips in the center of a table. Suppose there are two players who could alternately remove one, two or three chips from the pile. At least one chip must be removed, but no more than three chips can be removed in a single move. The player that removes the last chip wins. Does the first player (the player who starts playing the game) have a winning strategy in this game, that is, whatever moves his opponent makes, he can always make his moves in a certain way ensuring his win? Justify your answer. [8]

A2. Consider a snail at the foot of a pole that is h metres high. The snail starts climbing up the pole at an initial speed of c metres/hour. After every d hours of climbing, the snail takes a nap for z hours. While it is asleep, it slips down the pole at s metres/hour. After waking up from the nap, it continues climbing, but its speed reduces by $p\%$ of the speed that it had just before it took this nap.

In your answer booklet, write the expressions/conditions corresponding to the blanks in the pseudo-code below, so that the code determines the total number of hours it takes the snail to reach the top of the pole. If the snail can never reach the top of the pole, the code should print an appropriate message. [6]

Step 1. Set `remainingHeight` to h , `speed` to c , and `numHours` to 0.

Step 2. If _____, print message:
“*Snail reaches the top in _____ hours*”
and STOP.

Step 3. Otherwise, increment `numHours` by _____;
decrement `remainingHeight` by _____.

Step 4. If _____, print message:
“*Snail will never be able to reach the top*”
and STOP.

Step 5. Otherwise, set speed to _____,
and repeat from Step 2.

A3. The integers 1, 2, 3, 4 and 5 are to be inserted into an empty stack using the following sequence of PUSH() operations:

PUSH(1) PUSH(2) PUSH(3) PUSH(4) PUSH(5)

Assume that POP() removes an element from the stack and outputs the same. For each of the following output sequences, determine whether it can be generated by inserting suitable POP() operations into the above sequence of PUSH() operations. Justify your answers.

(a) 1 2 3 4 5

(b) 5 4 1 2 3. [6]

A4. Consider an array $X = [x_1, x_2, \dots, x_n]$ of n real numbers sorted in ascending order as input. Another array $Y = [y_1, y_2, \dots, y_n]$ is created such that $y_i = x_i^2 + x_i + 1$. Write pseudocode for an efficient algorithm to sort Y . (You will get full credit if your algorithm uses no more than kn comparisons, where k is some constant.) [6]

A5. Let M be a 4-digit positive integer. Let N be the 4-digit integer obtained by writing the digits of M in reverse order. If $N = 4M$, then find M . Justify your answer. [6]

A6. Suppose there are five pairs of shoes in a closet, from which four shoes are taken out at random. Calculate the probability that among these four shoes, there is at least one complete pair. [5]

A7. Let $P(x), Q(x)$ be two distinct polynomials (over \mathbb{R}) with degree of P and Q being at most $(d - 1)$. Let a_0, \dots, a_{n-1} be distinct elements of \mathbb{R} . Prove that $|\{i : P(a_i) \neq Q(a_i)\}| \geq n - d + 1$. [6]

A8. Let A be an $n \times n$ integer matrix whose columns are pairwise distinct. Show that there is a row in A , whose removal results in an $(n - 1) \times n$ matrix with the same property. [6]

- A9. Let G be a finite group. Assume that G has exactly one element a of order 2. Then show that

$$\prod_{g \in G} g = a.$$

[5]

- A10. Evaluate

$$\lim_{x \rightarrow 0} \left(x^2 \left(1 + 2 + 3 + \cdots + \left[\frac{1}{|x|} \right] \right) \right)$$

For any real number a , $[a]$ is the largest integer not greater than a . [6]