- The symbol $\mathbb{R}$ denotes the set of all real numbers.
- The symbol $\mathbb{Q}$ denotes the set of all rationals numbers.
- The symbol $\mathbb{Z}$ denotes the set of all integers.
- The symbol $\mathbb{N}$ denotes the set of all natural numbers.

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1. Which of the inferences are valid given the premises?

(I_1)
All birds have feathers.
Robins are birds.
Therefore, robins have feathers.

(I_2)
All insects need oxygen.
Mosquitoes need oxygen.
Therefore, mosquitoes are insects.

(I_3)
All flowers need water.
Roses need water.
Therefore, roses are flowers.

(I_4)
All mammals walk.
Whales are mammals.
Therefore, whales walk.

(A) I_1, I_2 and I_4 (B) I_1 and I_4
(C) I_1, I_2 and I_3 (D) I_1 and I_3

2. The number of factors of 2025 (including 1 and itself) equals

(A) 15 (B) 25 (C) 35 (D) 45

3. Suppose the function `f` is defined by the following pseudocode. What will be printed when `f(5)` is called?

```
function f(x) {
    if (x > 1) {
        print(x);
        f(x-2);
        print(x);
    }
}
```

(A) 5 5 3 3 (B) 5 3 5 3 (C) 5 5 5 5 (D) 5 3 3 5

4. Consider the following statements.

   S1: In every country, there is a city through which a river passes.

   S2: There is no country in which every city has a river passing through it.

   S3: There is no country in which there is no city through which a river passes.

   S4: In every country, every river passes through a city.

   Which of the above statements are equivalent to each other?

   (A) S1 and S2 (B) S1 and S3 (C) S2 and S4 (D) S2 and S3

5. If the matrix

$$A = \begin{bmatrix} a & 1 \\ 2 & 3 \end{bmatrix}$$

has 1 as an eigenvalue, then $\det(A)$ equals

(A) 1 (B) 2 (C) 4 (D) 10

6. Let A be the following 4×4 matrix:

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}.$$

The rank of A equals

(A) 1 (B) 2 (C) 3 (D) 4

7. The remainder obtained when $\sum_{m=1}^{2025} m!$ is divided by 18 is

(A) 7 (B) 9 (C) 12 (D) 16

8. Consider the equation $x^7 + 4x^5 + 2x^3 + x + 1 = 0$. The number of real root(s) of this equation is

(A) 1 (B) 3 (C) 5 (D) 7

9. Let Z be the number of heads obtained when Alice tossed an unbiased coin 4 times. Then, Bob tossed the same coin a further Z times. Given that Bob got 2 heads, what is the probability that $Z = 3$?

(A) 4/9 (B) 1/4 (C) 2/9 (D) 1/2

10. Let R be the region defined by

$$R = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 50, x + y \geq 0\}.$$

The area of R equals

(A) 10π (B) 15π (C) 20π (D) 25π

11. For functions $f, g : \mathbb{N} \to \mathbb{R}$, we say that f is $o(g)$ (in words, f *is little-oh of* g) if $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$. Then

(A) n is $o\left(n^{1/2}\right)$ (B) $n^{-1/2}$ is $o\left(n^{-1/4}\right)$

(C) $n^{-1/4}$ is $o(n^{-1/2})$ (D) 1 is $o\left(n^{-1/2}\right)$

12. Let A be a 4×4 upper triangular matrix whose diagonal entries are $-1, 1, -\frac{1}{2}, \frac{1}{2}$. Then A^{-1} equals

(A) $5A - 4A^3$ (B) $5A + 4A^3$

(C) $-5A + 4A^3$ (D) $-5A - 4A^3$

13. If α, β, γ are roots of $x^3 + ax + 1 = 0$, then

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + \frac{\beta}{\gamma} + \frac{\gamma}{\beta} + \frac{\alpha}{\gamma} + \frac{\gamma}{\alpha}$$

equals

(A) a (B) $3a$ (C) -1 (D) -3

14. Let $f : [0,1] \to [0,1]$ be defined as follows. For $x \in [0,1]$,

$$f(x) = \begin{cases} \frac{x}{2} & \text{if } x \text{ is rational,} \\ \frac{x+1}{2} & \text{if } x \text{ is irrational.} \end{cases}$$

(A) f is a bijection.

(B) f is neither injective, nor surjective.

(C) f is injective, but not surjective.

(D) f is surjective, but not injective.

15. Suppose f is a real-valued function defined on the set of real numbers. Let

$$f(x) = \frac{\sin x}{x} \quad \text{for } \ x \neq 0, \ \ f(0) = a,$$

where a is a real number. Let f be differentiable at $x = 0$. Let $f'(0) = b$. Then $a + b$ equals

(A) -1 (B) 0 (C) 1 (D) 2

16. There are 10 points on a plane in such a way that 6 of these points are collinear and other than these 6 points, no set of 3 or more points are collinear. The number of triangles which can be formed with vertices among these 10 points equals

(A) 10 (B) 60 (C) 100 (D) 120

17. We call a function $\phi : \mathbb{R} \longrightarrow \mathbb{R}$ a homomorphism if

$$\phi(x + y) = \phi(x) + \phi(y) \text{ for all } x, y \in \mathbb{R}.$$

Let

$$f(x) = 2x, \ \ g(x) = x + 1 \ \text{ and } \ h(x) = 2x + 1.$$

Which of the following statements is correct?

(A) f is a homomorphism but g, h are not.
(B) g is a homomorphism but f, h are not.
(C) f and g are homomorphisms but h is not.
(D) f, g, h are all homomorphisms.

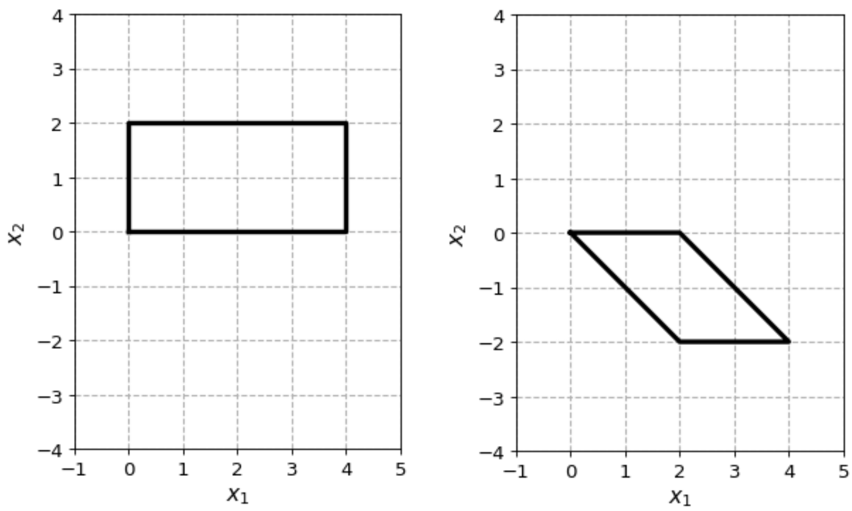
18. Let $L = (a + b)(a + b)^*a(a + b)^*$ be a regular language over $\{a, b\}$. Which of the following strings does not belong to L?

(A) aaa (B) bba (C) abb (D) aab

19. In the decimal system with digits in the set $\{0, \ldots, 9\}$, a notation like 234 stands for $2 \times 10^2 + 3 \times 10 + 4$; but in a different base r with digits in the set $\{0, \ldots, r-1\}$, the notation 234 will stand for the number $2 \times r^2 + 3 \times r + 4$. If the relation $430 + 240 = 1000$ holds when numbers are represented in a certain base r, then r equals

(A) 5 (B) 6 (C) 7 (D) 8

20. Let R be the set of points (represented by the column vectors $(x_1\, x_2)^T$) on the 2-dimensional vector space $\mathbb{R}^2$, constituting the rectangle shown in the figure on the left. Let T be a linear transformation such that the parallelogram P shown in the figure on the right is obtained by transforming all points in R by T. In other words, $P = \{Tx : x \in R\}$.



Then, T can be represented by the matrix:

(A) $\begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix}$ (B) $\begin{bmatrix} 0.5 & 1 \\ 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 0.5 & 1 \\ 0 & -1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 \\ 0 & -0.5 \end{bmatrix}$

21. There are 4 suspects, A, B, C and D, say, in a criminal case. At least one of them is guilty. If A is guilty then so is B. If A and D are innocent, then so is C, and if A or C is innocent, then so is B. Suppose the police found out that B is guilty. From the given information, the persons who can be certainly declared guilty are:

(A) A, B, C and D (B) A, B and C
(C) B, C and D (D) A, B and D

22. Let `f(x)` be defined as follows:

```
function f(x) {
    count := 0;
    while (x > 0) {
        x := floor(x/2);
        count := count + 1;
    }
    return count;
}
```

where `floor(y)` is defined as the largest integer less than or equal to `y`. Suppose, for a positive integer `x`, `f(x) = 8`. What is the maximum possible value of `x`?

(A) 255 (B) 256 (C) 511 (D) 512

23. Twelve monkeys have collected a total of N bananas where every monkey has managed to collect at least one banana. What is the maximum value of N for which you can guarantee that at least two of the monkeys have gathered the same number of bananas?
(A) 72 (B) 77 (C) 84 (D) 88

24. Five marbles of different sizes are to be coloured using the three colours, namely red, yellow and blue, where each marble is coloured with exactly one colour. The number of possible colourings of all the marbles such that each colour is used to colour at least one marble is

(A) 32 (B) 64 (C) 75 (D) 150

25. Consider the real-valued function

$$f(x) = x + \frac{1}{x},\ x > 0.$$

The tangents to the graph of f at the points $(1/2, f(1/2))$ and $(2, f(2))$ intersect at the point $(4/5, 8/5)$. The area of the region enclosed by the graph of f and these two tangents equals

(A) $2 \log 2 + \frac{15}{8}$ (B) $2 \log 2 - \frac{6}{5}$

(C) $2 \log 2 + \frac{15}{4}$ (D) $2 \log 2 - \frac{3}{5}$

26. A coin with probability of head equal to $1/3$ is repeatedly tossed until at least one head and at least one tail is obtained. The random experiment is stopped as soon as this condition is satisfied. The probability that the experiment stops after exactly 8 tosses equals

(A) $\frac{128}{6561}$ (B) $\frac{130}{6561}$ (C) $\frac{192}{6561}$ (D) $\frac{256}{6561}$

27. Which of the following can be the degrees of the vertices of a 5-vertex undirected simple graph?

(A) $0, 1, 2, 3, 4$ (B) $0, 3, 3, 4, 5$
(C) $1, 1, 2, 2, 3$ (D) $2, 2, 3, 3, 4$

28. Consider the set of all possible polynomials

$$x^3 + a_2x^2 + a_1x + a_0,$$

with a_0, a_1, a_2 integers and $|a_0| \in \{0, \ldots, 9\}$, such that for each polynomial, all of its roots are distinct positive integers in some geometric progression. The number of such polynomials is

(A) 0 (B) 1 (C) 2 (D) 3

29. If two languages L_1 and L_2 are both non-regular, then which of the following is always true?

(A) $L_1 \cup L_2$ is regular.

(B) $L_1 \cap L_2$ is finite.

(C) Both L_1 and L_2 are infinite.

(D) $L_1 \setminus L_2$ is not regular.

30. The complement of a simple graph $G = (V, E)$ is defined as $\bar{G} = (V, \bar{E})$ such that for all $u, v \in V$, $uv \in E \iff uv \notin \bar{E}$. Let G be a graph on n verties such that $\bar{G}$ is isomorphic to G. Then, which of the following is necessarily true?

(A) $n \equiv 0 \pmod 4$ or $n \equiv 1 \pmod 4$

(B) $n \equiv 0 \pmod 4$ or $n \equiv 2 \pmod 4$

(C) $n \equiv 1 \pmod 4$ or $n \equiv 2 \pmod 4$

(D) $n \equiv 2 \pmod 4$ or $n \equiv 3 \pmod 4$