1. (a) If a, b, c are positive real numbers and $a \neq b \neq c$, then show that the value of the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is negative.

   (b) Three real numbers are in Arithmetic Progression, the first term is 9. If 2 is added to the second term and 20 is added to the third term, the resulting three numbers are in Geometric Progression. What is the smallest possible value for the third term of the Geometric Progression? $[8 + 7 = 15]$

2. (a) The bleaching stage of a pulp manufacturing process needs to be executed in such a way to achieve a pulp brightness of 50, pulp whiteness of 80 and viscosity of 30. This can be done by controlling the Chemical Flow, Duration and Temperature of the bleaching process. The equations depicting the relationship between Brightness, Whiteness and Viscosity with the Chemical flow, Duration and Temperature are given below. Determine the levels of Chemical flow, Duration and Temperature required to be maintained during the bleaching process to achieve the required brightness, whiteness and viscosity.
   Brightness = 2 x Chemical Flow - 15 x Duration + Temperature
   Whiteness = Chemical Flow - 25 x Duration + 2 x Temperature
   Viscosity = 5 x Chemical Flow + 20 x Duration - Temperature

   (b) There are 4 Red, 5 Black and 6 Blue balls in an urn. 4 balls are drawn at random without replacement from this urn. What is the probability that these 4 balls will contain at least one ball of each colour? $[6 + 9 = 15]$

3. (a) Evaluate: $\int \frac{(\sqrt{x})^5}{(\sqrt{x})^7 + x^6} dx$

   (b) Solve the differential equation
   $(xdy - ydx)\sin(\frac{y}{x})y = (ydx + xdy)x\cos(\frac{y}{x}), x \neq 0.$ $[7 + 8 = 15]$

4. (a) The length of a side of a square is 'a' metres. A second square is formed by joining the middle points of the sides of this square. Then, a third square is formed by joining the middle points of the second square and so on. Calculate the sum of the area of all squares which are formed in the way as mentioned above.

   (b) If $x^2 - 5x + 1 = 0$, find the value of $\frac{x^{10}+1}{x^5}$. $[8 + 7 = 15]$

5. (a) Let $A = \lim_{n\to\infty} \left\{ \left(\sin \frac{\pi}{2n}\right) \times \left(\sin \frac{2\pi}{2n}\right) \times \left(\sin \frac{3\pi}{2n}\right) \times \ldots \times \left(\sin \frac{n\pi}{2n}\right) \right\}^{\frac{1}{n}}$. Find the value of A.

(b) Let $\Delta_a = \begin{vmatrix} a-1 & n & 6 \\ (a-1)^2 & 2n^2 & 4n-2 \\ (a-1)^3 & 3n^3 & 3n^2-3n \end{vmatrix}$, find the value of $\sum_{a=1}^{a=n} \Delta_a$. [9 + 6 = 15]

6. (a) Let $f(x) = \sum\limits_{n=0}^{\infty} \frac{x^n}{n!2^{n(n-1)/2}}$. Then find the value of $\lim\limits_{x\to 0} \frac{f(x)-e^x}{1-\cos x}$.

(b) If $a_1, a_2, a_3, ..., a_n$ are in Harmonic Progression ($a_i \neq 0$, for all $i = 1, 2, ..., n$), then show that

$$a_1a_2 + a_2a_3 + a_3a_4 + ... + a_{n-1}a_n = (n-1)a_1a_n, n \geq 3.$$

[8 + 7 = 15]

7. (a) Evaluate $\int_1^2 [x^2]dx$, where $[x]$ is the greatest integer less than or equal to x.

(b) A biased coin has probability p of coming up heads on any toss. Thus $P(H) = p$. When it is tossed twice, it is known that

$$P(HTorHH) = \frac{5}{6}.$$

If the same coin is tossed three times, then find the probability of observing exactly two heads. [8 + 7 = 15]

8. (a) Solve the equation: $3^{\sin 2x + 2\cos^2 x} + 3^{1-sin2x+2\sin^2 x} = 28$.

(b) Let $a_0 = a_1 = 1$. For $n = 1, 2...$ define $a_{n+1} = (1 + a_n + a_{n-1}^2)/4$. Does the sequence $\{a_n\}$ converge to a finite limit as $n \to \infty$? Find the limit if it does. [8 + 7 = 15]