1. Consider a monopoly firm facing a market demand function $p = 12 - q$, where p is price, and q is quantity. The monopolist has a single plant that can produce an output of q at a cost of $C(q)$, where $C(q) = 2 + \frac{q^2}{2}$, if $q > 0$, and $C(q) = 0$ otherwise.

   (a) Find the optimal monopoly output. What is the deadweight loss, and social welfare at the optimal monopoly output?

   (b) Suppose the monopoly firm can price discriminate perfectly, and can also sell in the world market for a constant price of 8. Solve for the optimal monopoly outcome.

   (c) Next suppose the monopoly firm has access to two plants, 1 and 2. Plant i, $i = 1, 2$, can produce an output of q_i at a cost of $q_i^2/2$.

      i. Solve for the firm's aggregate cost function, i.e. the total cost in case it decides to produce a total output of q, using either one, or both the plants. Use the aggregate cost function to solve for the optimal monopoly output.

      ii. What is the optimal monopoly outcome if the monopoly firm can price discriminate perfectly?

   $[8 + 6 + 10 = 24]$

2. An economy consists of two agents 1 and 2 and an initial endowment of Rs 100. A decision has to be made regarding the building of a bridge: $d = 1$ if the bridge is built and $d = 0$ if it is not. The bridge costs Rs 60 to build. If the bridge is built the remaining Rs 40 is distributed among the two agents; if it is not built, Rs 100 is distributed among the agents. An allocation in the economy consists of a triple (d, x_1, x_2) where $x_1 + x_2 = 40$ if $d = 1$ and $x_1 + x_2 = 100$ if $d = 0$. In each case $x_1, x_2 \geq 0$. Agent i, $i = 1, 2$ has a valuation $v_i \geq 0$ for the

bridge. Agent i's utility from the allocation (d, x_1, x_2) is $v_i d + x_i$.

(a) Suppose $v_1 = 25$, $v_2 = 45$. Is the allocation $(d = 0, 40, 60)$ Pareto-efficient? Justify your answer.

(b) Suppose $v_1 = 25$, $v_2 = 45$. Is the allocation $(d = 0, 70, 30)$ Pareto-efficient? Justify your answer.

(c) Show that if the allocation $(d = 1, x_1, x_2)$ is Pareto-efficient, then $v_1 + v_2 \geq 60$.

$[5 + 5 + 14 = 24]$

3. (a) An economist collects data from an experiment. Each data point is one of the two types: (i) `male` or (ii) `female`. The date is collected and sent in three boxes. One box contains data of *only* `male` type, another box contains data of *only* `female` type, and a third box contains data of *both* `male` and `female` type. When the three boxes reach the office of the economist, there are labels on each box (see Figure 1). The economist is told that the labels in *every* box is wrong.

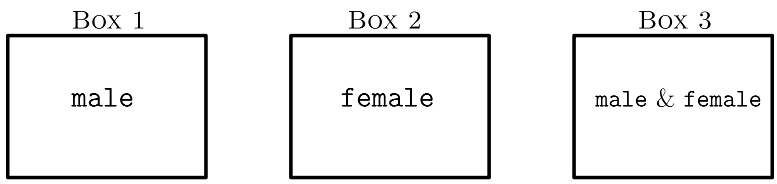

Figure 1: Each box is incorrectly labeled

The economist asks an MSQE student to sample data from the boxes and figure out the correct labels. What is the minimum number of samples that the MSQE student needs to draw to figure out the correct labels of all the boxes?

Describe your answer logically by showing which box(es) need to be sampled and how many times.

(b) Three students are standing in a straight line. Student 1 at the front, student 2 next, and student 3 at the last position. There are 3 RED hats and 2 BLUE hats. A teacher comes and puts a hat on each student. Suppose each student *only* sees the colour of the hats of students in front of her, but not her own hat or the hats of students behind her. So, student 3 sees the colour of the hats of student 1 and student 2; student 2 sees the colour of the hat of student 1; and student 1 does not see the colour of anyone's hat. Starting with student 3, followed by student 2, and finally student 1, each student is asked if she knows the color of her own hat. The students can either answer `yes` or `no`. Assume students answer truthfully and answer of each student is revealed to all the students.

i. What is the probability that student 3 says `no`?

ii. Suppose student 3 says `no`. What is the probability that student 2 says `no`? Note that student 2 knows the answer of student 3.

iii. Suppose student 3 and student 2 both say `no`. What is the probability that student 1 says `no`? Note that student 1 knows the answers of student 3 and student 2.

[5 + 18 = 23]

4. (a) How many real solutions does the following equation have?

$$(x^2 - 5x + 6)^{(x^2-7x+12)} = 1$$

(b) A function $f : \Re \rightarrow \Re$ is called **quasi-convex** if for every

$x, y \in \Re$ and every $\lambda \in [0,1]$, we have

$$f(\lambda x + (1-\lambda)y) \leq \max\{f(x), f(y)\}$$

i. Show that a convex function is quasi-convex.

ii. Show that a non-decreasing function is quasi-convex. Note that f is non-decreasing if $x > y$ implies $f(x) \geq f(y)$.

$[10 + 5 + 4 = 19]$

5. (a) Suppose that utility function is given by $u = \ln c + b\frac{(1-l)^{1-\gamma}}{1-\gamma}$ where c and l represent consumption and labour supply, and parameters b and γ are positive. Further, the wage rate per unit of labour supply is given by w. Agents consume out of their labour income which forms their budget constraint. If agents maximise utility subject to their budget constraint, how does labor supply depend on the real wage rate w. Clearly show all the derivations and explain the result.

(b) Now consider that agents live for two periods and they have the following utility function: $u = \ln c_1 + 2b\sqrt{(1-l_1)} + \beta[\ln c_2 + 2b\sqrt{(1-l_2)}]$ where c_i, l_i represent consumption and labour supply in period $i = 1, 2$ respectively, the parameter β is positive while the rest of the notation is the same as in part (a) above. Further, any consumption made or income earned in the second period is discounted at the rate, $1 + r$. Agents maximise their utility subject to the life-time budget constraint. Further, leisure in period $i = 1, 2$ is denoted by $q_i = 1 - l_i$.

i. Find out the expression for the ratio of the optimal $\frac{q_1}{q_2}$ in terms of the wage ratio, $\frac{w_2}{w_1}$, and other parameters of the model. How does l_1 vary with $\frac{w_2}{w_1}$? Explain the result.

ii. Let q^* be the ratio of optimal q_1 to q_2, that is, $\frac{1-l_1}{1-l_2} = q^*$ and $\frac{w_2}{w_1} = w^*$. Calculate the elasticity of q^* with respect to w^*. Interpret your result in a couple of sentences.

$[8 + 15 + 7 = 30]$