- The symbol $\mathbb{R}$ denotes the set of all real numbers.
- The symbol $\mathbb{Q}$ denotes the set of all rationals numbers.
- The symbol $\mathbb{Z}$ denotes the set of all integers.
- The symbol $\mathbb{N}$ denotes the set of all natural numbers.

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1. The number of factors of 2025 (including 1 and itself) equals

   (A) 15 (B) 25 (C) 35 (D) 45

2. If the matrix
$$A = \begin{bmatrix} a & 1 \\ 2 & 3 \end{bmatrix}$$
   has 1 as an eigenvalue, then $\det(A)$ equals

   (A) 1 (B) 2 (C) 4 (D) 10

3. Suppose $\sec x + \tan x = 3$ for some $x \in (0, \pi/2)$. Then $\sin 2x$ equals

   (A) $\frac{1}{25}$ (B) $\frac{3}{5}$ (C) $\frac{4}{5}$ (D) $\frac{24}{25}$

4. Let $V := \{a_0 + a_1x + a_2x^2 |\ a_0, a_1, a_2 \in \mathbb{R}\}$. Define a linear transformation $T : V \to V$ by $T(f) = f'$. Then the rank of T^3 is

   (A) 0 (B) 1 (C) 2 (D) 3

5. Consider the equation $x^7 + 4x^5 + 2x^3 + x + 1 = 0$. The number of real root(s) of this equation is

(A) 1 (B) 3 (C) 5 (D) 7

6. Let $i = \sqrt{-1}$ and ω be a non-real root of the equation $x^3 = 1$. If $(i\omega)^n$ is an integer, then n is necessarily

(A) a multiple of 3 but need not be even
(B) even but need not be a multiple of 3
(C) a multiple of 12
(D) a multiple of 6 but need not be a multiple of 12

7. Let G be a cyclic group of order 30 and let g be a generator of G. Then the order of g^{24} is

(A) 3 (B) 5 (C) 6 (D) 15

8. The remainder obtained when $\sum_{m=1}^{1000} m!$ is divided by 18 is

(A) 7 (B) 9 (C) 12 (D) 16

9. There are two types of newly launched phones A and B. The probability that type A has good battery life is 0.9 and the probability that type B has good battery life is 0.7. Suppose from a collection of phones containing equal number of phones of each type, a phone is selected at random. The probability that the selected phone has good battery life equals

(A) 0.63 (B) 0.75 (C) 0.80 (D) 0.85

10. Let R be the region defined by

$$R = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 50, x + y \geq 0\}.$$

The area of R equals

(A) 10π (B) 15π (C) 20π (D) 25π

11. For functions $f, g : \mathbb{N} \to \mathbb{R}$, we say that f is $o(g)$ (in words, f *is little-oh of* g) if $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$. Then

(A) n is $o\left(n^{1/2}\right)$ (B) $n^{-1/2}$ is $o\left(n^{-1/4}\right)$

(C) $n^{-1/4}$ is $o(n^{-1/2})$ (D) 1 is $o\left(n^{-1/2}\right)$

12. If α, β, γ are roots of $x^3 + ax + 1 = 0$, then

$$\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + \frac{\beta}{\gamma} + \frac{\gamma}{\beta} + \frac{\alpha}{\gamma} + \frac{\gamma}{\alpha}$$

equals

(A) a (B) $3a$ (C) -1 (D) -3

13. Let A be a 4×4 upper triangular matrix whose diagonal entries are $-1, 1, -\frac{1}{2}, \frac{1}{2}$. Then A^{-1} equals

(A) $5A - 4A^3$ (B) $5A + 4A^3$

(C) $-5A + 4A^3$ (D) $-5A - 4A^3$

14. Let $\alpha = \sin 10^\circ$. Then α is a root of the equation

(A) $8x^3 - 6x - 1 = 0$

(B) $8x^3 - 6x + 1 = 0$

(C) $8x^3 + 6x - 1 = 0$

(D) $8x^3 + 6x + 1 = 0$

15. Suppose f is a real-valued function defined on the set of real numbers. Let

$$f(x) = \frac{\sin x}{x} \quad \text{for } x \neq 0, \quad f(0) = a,$$

where a is a real number. Let f be differentiable at $x = 0$. Let $f'(0) = b$. Then $a + b$ equals

(A) -1 (B) 0 (C) 1 (D) 2

16. The equation

$$(x-2)^2 + (y-3)^2 + (x-2)(y-3) = 0$$

represents

(A) a parabola (B) an ellipse

(C) a pair of straight lines (D) a point

17. Suppose P is a moving point in the Euclidean plane whose distance from the point $A : (0, 0)$ is twice the distance from the point $B : (3, 0)$. The locus of P is

(A) a circle (B) a hyperbola

(C) an ellipse (D) a straight line

18. There are 10 points on a plane in such a way that 6 of these points are collinear and other than these 6 points, no set of 3 or more points are collinear. The number of triangles which can be formed with vertices among these 10 points equals

(A) 10 (B) 60 (C) 100 (D) 120

19. Consider the functions $\mathbb{R} \longrightarrow \mathbb{R}$ defined by

$$f(x) = 2x, \quad g(x) = x + 1 \text{ and } h(x) = 2x + 1.$$

Which of the following statements is correct?

(A) f is a group homomorphism but g, h are not
(B) g is a group homomorphism but f, h are not
(C) f and g are group homomorphisms but h is not
(D) f, g, h are all group homomorphisms

20. Let $f : [0, 1] \to [0, 1]$ be defined as follows. For $x \in [0, 1]$,

$$f(x) = \begin{cases} \dfrac{x}{2} & \text{if } x \text{ is rational,} \\ \dfrac{x+1}{2} & \text{if } x \text{ is irrational.} \end{cases}$$

(A) f is a bijection
(B) f is neither injective, nor surjective
(C) f is injective, but not surjective
(D) f is surjective, but not injective

21. In the decimal system with digits in the set $\{0, \ldots, 9\}$, a notation like 234 stands for $2 \times 10^2 + 3 \times 10 + 4$; but in a different base r with digits in the set $\{0, \ldots, r - 1\}$, the notation 234 will stand for the number $2 \times r^2 + 3 \times r + 4$. If the relation $430 + 240 = 1000$ holds when numbers are represented in a certain base r, then r equals

(A) 5 (B) 6 (C) 7 (D) 8

22. Consider the following two subsets of the group $\mathbb{Q}$ of rational numbers under addition:

(1) $H = \{\frac{m}{n} : m, n \in \mathbb{Z}, n \neq 0, n \text{ divides } 2025\}$,

(2) $K = \{\frac{m}{n} : m, n \in \mathbb{Z}, n \neq 0, n \text{ is co-prime to } 2025\}$.

Which of the following statements is correct?

(A) Both H and K are subgroups of $\mathbb{Q}$ under addition
(B) H is subgroup of $\mathbb{Q}$ under addition but K is not
(C) K is subgroup of $\mathbb{Q}$ under addition but H is not
(D) Neither H nor K is a subgroup of $\mathbb{Q}$ under addition

23. The equation of the curve satisfying

$$y\,dx - x\,dy + \log\, x\,dx = 0,\ x > 0,$$

and passing through $(1, -1)$ is

(A) $y + \log x + 1 = 0$

(B) $-y^2 + \log x + 1 = 0$

(C) $y^3 + (\log x)^2 + 1 = 0$

(D) none of the above

24. Let D be the triangular region whose vertices are $(0, 0), (0, 2), (2, 0)$. The integral

$$\iint_D xy\,dx\,dy$$

equals

(A) 4/3 (B) 2/3 (C) 1 (D) 1/2

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25. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function defined by

$$f(x,y) = \begin{cases} \dfrac{y}{|x|}\sqrt{x^2+y^2}, & x \neq 0, \\ 0, & x = 0. \end{cases}$$

Then

(A) f is not continuous, and the directional derivatives do not exist at $(0,0)$

(B) f is not continuous, but the directional derivatives exist at $(0,0)$

(C) f is continuous, but the directional derivatives do not exist at $(0,0)$

(D) f is continuous, and the directional derivatives exist at $(0,0)$

26. Consider the real-valued function

$$f(x) = x + \frac{1}{x}, \; x > 0.$$

The tangents to the graph of f at the points $(1/2, f(1/2))$ and $(2, f(2))$ intersect at the point $(4/5, 8/5)$. The area of the region enclosed by the graph of f and these two tangents equals

(A) $2\log 2 + \frac{15}{8}$ (B) $2\log 2 - \frac{6}{5}$

(C) $2\log 2 + \frac{15}{4}$ (D) $2\log 2 - \frac{3}{5}$

27. A coin with probability of head equal to $1/3$ is repeatedly tossed until at least one head and at least one tail is obtained. The random experiment is stopped as soon as this condition is satisfied. The probability that the experiment stops after exactly 8 tosses equals

(A) $\frac{128}{2187}$ (B) $\frac{130}{6561}$ (C) $\frac{256}{2187}$ (D) $\frac{128}{6561}$

28. Five marbles of different sizes are to be coloured using the three colours red, yellow, and blue, where each marble is coloured with exactly one colour. The number of possible colourings of all the marbles such that each colour is used to colour at least one marble is

(A) 32 (B) 64 (C) 75 (D) 150

29. Consider the set of all possible polynomials

$$x^3 + a_2x^2 + a_1x + a_0,$$

with a_0, a_1, a_2 integers and $|a_0| \in \{0, \ldots, 9\}$, such that for each polynomial, its roots are distinct positive integers in some geometric progression. The number of such polynomials is equal to

(A) 0 (B) 1 (C) 2 (D) 3

30. Let

$$A = \{x \in [0, \infty) : \text{the series} \sum_{k=1}^{\infty} (x)^{k^{1/k}} \text{converges}\}.$$

Then

(A) $A = \{0, 1/2\}$ (B) $A = [0, 1)$

(C) $A = \{0\}$ (D) $A = [0, 1/2)$