1. Ahmed is currently spending his income to maximize his satisfaction. He is renting an apartment for ₹ 9000 per month as shown in the graph below in Figure 1. (Assume each rupee spent on housing buys one unit of housing). His current equilibrium or optimal choice is represented by point E.

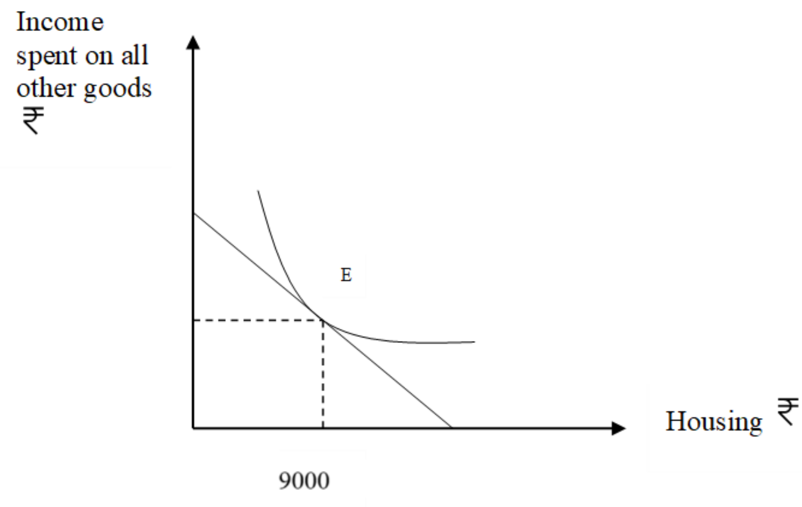

Figure 1: Ahmed's equilibrium.

   (a) Suppose Ahmed qualifies for a government housing assistance program that will provide him with a ₹ 6000 per month apartment at no charge. If he accepts the apartment, he cannot augment expenditure on housing beyond ₹ 6000 nor can he exchange the apartment for cash (rent it out) or other goods. Depict graphically Ahmed's budget line if he accepts this government offer. Explain your answer.

   (b) Suppose the government instead gives cash of ₹ 6000 under the housing scheme (and not an apartment worth ₹ 6000 per month). How will this affect Ahmed's budget line? Explain and show your answer in a graph.

(c) Is Ahmed indifferent between the two housing programs (a and b) or does he prefer one to the other? Draw indifference curves to illustrate and explain your answer.

$[5 + 5 + 5]$

2. Consider a city with n people in which each person gets a welfare benefit of $U(G)$ from having G hectares of public parks in the city. Suppose the cost of providing each hectare of parks is one lakh rupees. The net utility of person i who pays c lakh rupees for park provision is $U(G)-c$ when there are G hectares of public parks in the city. U is strictly increasing and strictly concave, and $U^{'}(G)$ approaches ∞ as G approaches zero from the right.

   (a) If a utilitarian social planner decides the level of park provision G^* with the cost being financed by equal per capita taxes, what condition must G^* satisfy?

   (b) Suppose instead that each person in the city voluntarily decides how much to contribute to a fund that will be used for park provision, taking all other persons' contributions as given. What condition must the resulting level of park provision G_v satisfy? How does it compare to the optimum of the social planner?

   (c) Now suppose that the level of park provision is decided by voting. Each person votes for their own most preferred level of provision knowing that the cost will be equally shared among the residents of the city. The median voter's choice will be implemented. What condition must the resulting level of provision G_m satisfy? How does it compare to the optimum G^* of the social planner?

   (d) Now suppose instead that one-third of the voters each get a welfare benefit of $3U(G)$ from G hectares of parks, while the remaining two-thirds each get a welfare benefit of $U(G)$ as before. Costs are as before. Now compare the social planners' solution with the one obtained under median voting.

$[5 + 5 + 5 + 5]$

3. Consider an economy where the agents are identical. An infinitely-lived representative agent maximizes her life-time utility

$$\sum_{t=0}^{\infty} \beta^t u(c_t, m_t)$$

where $0 < \beta < 1$ is a parameter, u represents utility per period that depends on consumption c_t and real per-capita money holding m_t in period t. Agents choose consumption and real money holding to maximize their life-time utility (or welfare). Marginal utility from both consumption and real money holding is positive. Each agent has the following budget constraint in each period t:

$$y_t + \tau_t + \frac{(1 + i_{t-1})b_{t-1}}{1 + \pi_t} + \frac{m_{t-1}}{1 + \pi_t} + (1 - \delta)k_{t-1} = c_t + m_t + k_t + b_t$$

where y_t denotes individual income, k_t represents capital chosen in period t, b_t denotes the amount spent on a one-period bond purchased in period t and τ_t denotes transfers received in period t. As usual, the subscript t denotes values in period t. Suppose output is produced using a strictly concave differentiable production function f where $y_t = f(k_{t-1})$ which means output y_t in period t is produced with the capital available in period $t - 1$, k_{t-1}. $1 + i_{t-1}$ represents the total interest earned on one unit of bonds purchased in period $t - 1$ maturing in period t. The inflation rate in period t is represented by π_t. Additionally, the parameter $0 < \delta < 1$ represents a period-wise constant depreciation rate of capital.

The Lagrangean for an agent's maximization problem is as follows:

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_t, m_t) + \lambda_t \left(\xi_t + (1-\delta)k_{t-1} - c_t - m_t - k_t - b_t \right) \right],$$

where

$$\xi_t = y_t + \tau_t + \frac{(1+i_{t-1})b_{t-1}}{1+\pi_t} + \frac{m_{t-1}}{1+\pi_t},$$

λ_t is the Lagrange multiplier in t and the agent chooses c_t, m_t, k_t and b_t.

(a) Write down the first-order conditions for the maximization problem given above.

(b) Using the first-order conditions derived in the previous part, show the relationship between interest on bonds i_t, inflation π_{t+1}, and the real rate of interest r_t where $r_t = f'(k_{t-1}) - \delta$.

[20 + 10 = 30]

4. From my bedroom window, if the air quality is bad enough, I can't see the Indian flag flying over the ISI campus. I run the following regression:

$$\text{AQI}_d = \alpha + \beta \times I(\text{Flag is visible})_d + \epsilon_d$$

where d records which day of the year the observation is recorded, AQI is the air quality index, I(Flag is Visible) is an indicator variable for whether the flag is visible:

$$I(\text{Flag is visible}) = \begin{cases} 1, & \text{if flag is visible} \\ 0, & \text{if flag is not visible} \end{cases}$$

and ϵ is the error term. A higher AQI indicates worse air quality.

(a) How would you interpret the estimate of β?

(b) Under what conditions would knowing the estimate of β be useful to someone deciding whether or not to wear a mask that day? Explain your answer.

[5 + 10 = 15]

5. Suppose you want to study the effect of years of education x on income y:

$$\ln y_i = \beta_0 + \beta_1 x_i + \epsilon_i.$$

Assume that $E(\epsilon|x) = 0$. You estimate a relationship between education and income for the full sample as shown in Figure 2. You also run the same regression on a truncated sample in which (x, y) is observed iff $y < y_{poverty}$ where $y_{poverty}$ is the poverty line. This is depicted in Figure 3.

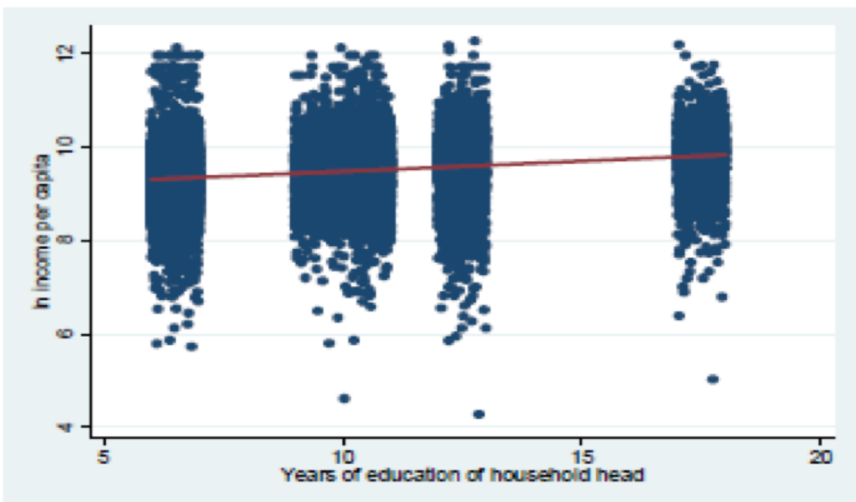

Figure 2: Full sample

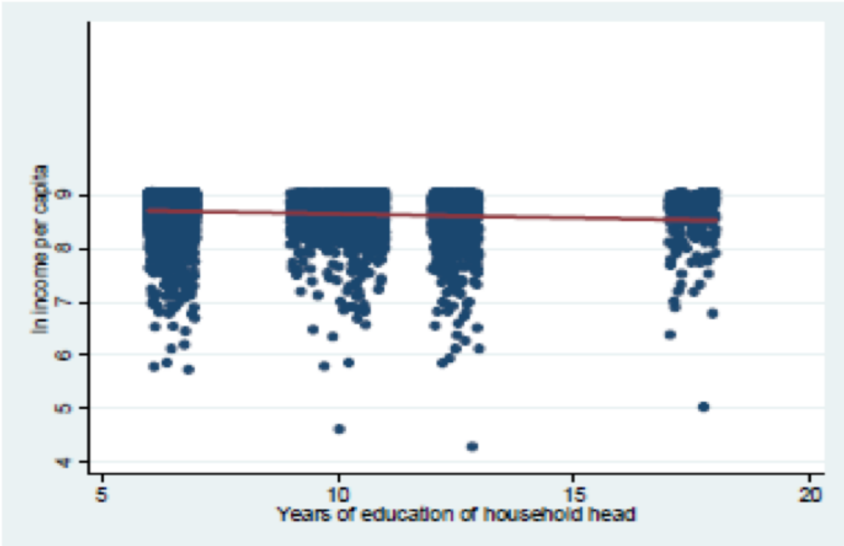

Figure 3: Below Poverty Line sample.

(a) Is there any difference between the estimates of β_1 you have obtained as shown in Figures 2 and 3? If yes, why? Explain your answer.

(b) Formally derive the relationship between education and income in the truncated sample, that is, derive the expected value of the log of income conditional on observing x and y only when $y < y_{poverty}$.

$[10 + 10 = 20]$