

## Group A

1. (i) Consider a competitive consumer who lives in a world with only two goods,  $X$  and  $Y$ ; the (non-negative) amounts of the two goods consumed are denoted by  $x$  and  $y$ , respectively. The consumer's preferences can be represented by a utility function  $U(x, y)$ ;  $U$  is continuous, twice differentiable, strictly quasi-concave and non-decreasing in its arguments. Let  $M$  denote the consumer's income and let  $p_i$  denote the price of good  $i$ ;  $i = X, Y$  and  $M, p_X, p_Y > 0$ . Suppose that the consumer spends a fixed proportion of her budget on  $X$ . Thus, for every price-income combination,  $p_X x = kM$ , where  $k$  is a constant,  $k \in (0, 1)$ . Prove that the consumer's demand behaviour can be rationalized in terms of utility maximization according to  $U$  subject to her budget constraint *only if*  $U$  is some positive monotone transformation of the linearly homogeneous Cobb-Douglas functional form  $x^\alpha y^{1-\alpha}$ .

(ii) Consider a competitive profit-maximizing firm producing a single output according to the production function

$$X = (a + b)^\gamma - b^2;$$

where  $X$  is the level of output,  $a$  and  $b$  are the (non-negative) amounts of the two inputs, and  $\gamma \in (0, 1)$ . Let  $P_a$  and  $P_b$  denote the respective input prices, and let  $P_X$  be the output price. Suppose that prices are initially such that the firm's profit-maximizing input combination involves positive amounts of both inputs.

- (a) Show that a marginal increase in  $P_b$  from such an initial situation must *increase* the firm's output.
- (b) Now consider the firm's cost minimization problem, subject to an exogenous output target,  $X^0$ . Assume, as before, an initial equilibrium where the firm's profit-maximizing input combination involves positive amounts of both inputs. Prove that a marginal increase in the firm's output target from such an initial equilibrium must reduce its cost-minimizing level of  $b$ .

[10+(8+7) = 25]

2. Consider a monopolist facing two consumers, A and B. All agents live for two periods – 1 and 2. At the beginning of period 1, the monopolist holds a stock of 2 units of a good which does not depreciate and lasts indefinitely. In their lifetime each consumer can consume at most one unit of the product. Therefore, if a consumer buys the product in period 1, she can use it for two periods, but if she buys the product in period 2, she uses it only for the second period. Notice that the monopolist's marginal cost of supplying the good, up to 2 units, is 0, as is its holding cost.

Let the money value of the per period benefit from using one unit of the product be  $v_A$  and  $v_B$  for consumer A and consumer B respectively. Assume that  $v_A > v_B > 0$ . Further, let  $\delta \in (0, 1]$  be the common time discount factor for all consumers and the monopolist. The monopolist knows the values of  $v_A$  and  $v_B$ , but cannot distinguish between the consumers on the basis of their valuations. Hence the two consumers cannot be charged different prices.

- (i) Suppose the monopolist rents out the product to the users for one period only – at the end of each period the monopolist gets the product back, which may again be rented out the next period. Let  $R_t$  be the per unit rent charged by the monopolist for the period  $t$  ( $t = 1, 2$ ). Find the profit-maximizing values of  $R_1$  and  $R_2$ . Find also the corresponding total discounted profit of the monopolist.
- (ii) Now suppose the monopolist is forced to sell the product **only** in period 1. Find the optimal price ( $P$ ) to be charged. Find also the corresponding monopoly profit.
- (iii) Next, suppose the monopolist can sell in either period, but must pre-commit to the period-specific prices. Thus, the monopolist declares the prices for the two periods,  $P_1$  and  $P_2$ , at the beginning of period 1, and cannot change them afterwards. The consumers know this and can choose to buy in either period (or not buy at all), at that period's prevailing price, declared at the outset. Suppose  $v_A > 2v_B$ . Find the two-period price combination that maximizes total (present discounted) profit. Construct the complete strategy profile that would sustain this profit-maximizing price combination as part of a Nash equilibrium.

- (iv) Lastly, suppose the monopolist can sell in either period, but cannot pre-commit to the period-specific prices. Thus, the monopolist declares the price for any period  $t$ ,  $P_t$ , only at the beginning of that particular period. In each period, the consumers choose whether to buy in that period subsequent to the declaration of that period's price. Suppose  $v_A > 2v_B$ . Explain intuitively whether the two-period price combination that maximizes total (present discounted) profit in part (iii) above can now be sustained as part of a subgame-perfect Nash equilibrium. If not, identify some other price combination be so sustained, intuitively justifying your claim.

[4+4+12+5 = 25]

### Group B

3. Consider an economy that can potentially accommodate money. There is one single good, and the nominal price of this good in period  $t$  is given by  $P_t$ . This economy is inhabited by agents who live for two periods, but the economy lives for eternity. These agents are each endowed with  $Y$  (a positive constant) units of the good in the first period when they are young and earn nothing in the second period of their life, when they are old. These agents leave no bequest nor inherit any. The total stock of the good in each period is fully exhausted through distribution among the young of that period.

At any time  $t$ , the population consists of  $N$  young agents and  $N$  old agents;  $N$  is normalized to unity. Agents have a two-period utility function given by

$$U = \ln c_t^y + \frac{1}{1+\rho} \ln c_{t+1}^o,$$

where  $c_t^y$  and  $c_{t+1}^o$  denote the consumption of an agent who is young in period  $t$  and her consumption in period  $t + 1$  (when she is old) respectively;  $\rho > 0$ . Agents can potentially carry over their first period endowments of the good to the second period through a storage technology, in which case they will get in the second period  $\frac{1}{1+\delta}$  times what they carry over from the first period, where  $\delta > -1$ . A young agent in period  $t$  can either consume her endowment in youth ( $c_t^y$ ) or store part of it  $S_t$ , (in which case  $\frac{S_t}{1+\delta}$  is available to her in period  $t + 1$ ) or trade it for

money  $M_t^d$ , where  $M_t^d$  denotes her demand for money. Thus, the agent has  $\frac{M_t^d}{P_t}$  amount of real balances at the end of period  $t$ . The budget equation of an agent who is young in period  $t$ , for that period, is accordingly given by:

$$Y = C_t^y + S_t + \frac{M_t^d}{P_t}.$$

An old agent gets a money transfer from the government - the real transfer to an old agent in period  $t + 1$  implied by that money transfer is denoted by  $T_{t+1}$ . Therefore,

$$c_{t+1}^o = \frac{S_t}{1+\delta} + \frac{M_t^d}{P_{t+1}} + T_{t+1}.$$

- (i) Show that, if the rate of inflation ( $\pi_t \equiv \frac{P_{t+1} - P_t}{P_t}$ )  $< \delta$  for all  $t$ , then young agents will never use the storage mechanism to carry any portion of their first period endowment into the next period.
- (ii) Assuming that  $\pi_t < \delta$ , solve for  $C_t^y$  and  $\frac{M_t^d}{P_t}$  in terms of  $Y$  and  $T_{t+1}$ .
- (iii) Assume that the money transfer from the government is financed through seigniorage:  $T_{t+1} = \frac{M_{t+1} - M_t}{P_{t+1}}$ , where  $M_{t+1}$  and  $P_{t+1}$  denote, respectively, the money supply and price level in period  $t + 1$ . Further suppose that the government follows a simple monetary rule given by  $M_{t+1} = (1 + \alpha)M_t$ , with  $\alpha > 0$ . Show that, in such a case where money is held in equilibrium (i.e.,  $\pi_t < \delta$  and money demand is equal to money supply): (a) the equilibrium rate of inflation is equal to the growth rate of money supply ( $\alpha$ ), and (b) equilibrium real consumption in old age ( $c_{t+1}^o$ ) falls if money supply grows faster (i.e., if the government increases  $\alpha$ ).

[5+8+(6+6) = 25]

4. Consider a macroeconomic model with three agents: an aggregate profit-maximizing firm, an aggregate household and a government. The firm produces a single final good, the output of which is denoted by  $Y$ , using a production function

$Y = F(L)$ ; where  $F' > 0$ ,  $F'' < 0$ . The price of this final good is denoted by  $P$ . The household's utility function is given by

$$U = \log C^\alpha + \log \left( \frac{M}{P} \right)^{1-\alpha} - V(L);$$

where  $0 < \alpha < 1$ ;  $C$  denotes the real consumption of the final good,  $M$  denotes the nominal money demand,  $L$  denotes the labour supplied by the household and  $V(L)$  is the disutility from labouring, with  $V' > 0$ ,  $V'' > 0$ . The household's budget constraint is given by

$$PC + M = wL + \pi + \bar{M} - PT;$$

where  $w$  is the wage rate,  $\pi (= PY - wL)$  is the nominal profit of the firm which is fully redistributed back to the household,  $\bar{M}$  is the initial endowment of money with the household and  $T$  is the real value of taxes levied by the government on the household. The government also makes a real expenditure of value  $G$  (which is another component of demand for the final good). The budget constraint of the government is given by

$$M - \bar{M} = P(G - T).$$

$G$  and  $T$  are exogenously given and  $G = T$ . Equilibrium in the final good market is given by:

$$Y = C + G.$$

The firm's equilibrium level of labour demand is given by

$$PF'(L) = w.$$

With the above given description of the economy, consider a scenario where price and wage are fully flexible, ensuring clearance of all markets.

- (i) Show that the equilibrium levels of output and employment ( $Y$  and  $L$ ) are determined independently of money ( $\bar{M}$ ).
- (ii) Derive an expression for the equilibrium price ( $P$ ) and show that it is proportional to  $\bar{M}$ .

[15+10 = 25]

### Group C

5. (i) Let  $\{Y_t\}$  be a time series process given by

$$Y_t = \varphi Y_{t-4} + \varepsilon_t,$$

where  $\varepsilon_t$  follows a white noise process with mean 0 and variance  $\sigma^2$ , and  $|\varphi| < 1$ .

(a) Is the process  $\{Y_t\}$  covariance-stationary? Prove your claim.

(b) Find the auto-correlation function of the process  $\{Y_t\}$ .

(ii) Now suppose

$$Y_t = \varphi Y_{t-4} + \varepsilon_t + \theta \varepsilon_{t-1},$$

where  $\varepsilon_t$  follows a white noise process with mean 0 and variance  $\sigma^2$ ,  $|\varphi| < 1$  and  $|\theta| < 1$ . Find the auto-correlation function of the process  $\{Y_t\}$ . Draw the correlogram plot for this process when  $\varphi = 0.6$  and  $\theta = -0.4$ .

[2+8+(10+5) = 25]

6. Consider the following about families in the United States:

- Marriages are unstable, and divorce is common. About half the marriages end in divorce.
- As a consequence, many children (0-15 year olds) see their parents divorce while they are in their teens.
- Marriages of “less than college-educated” couples are more unstable than marriages of “college-educated” couples. (Assume that spouses have identical educational attainments.)

Suppose you are an applied econometrician, and that you have information on the performance of children in high school (as measured by their math scores). You also have information on the divorce history of their mothers. (Assume that all children are born to mothers who were married.) You are interested in measuring

the true effect of mother's divorce on a child's math scores in school. Suppose you run the following regression:

$$\text{Test score}_i = \beta_0 + \beta_1 MD_i + \epsilon_i$$

where  $i$  refers to a child and  $MD_i$  is a dummy variable that takes the value 1 if  $i$ 's mother is divorced, and zero otherwise. Based on the information mentioned above, answer the following questions.

- (i) Is the estimate of  $\beta_1$  obtained from running the regression specified above a good estimate of the true effect of mother's divorce on a child's grade? If not, is  $\beta_1$  possibly an overestimate or an underestimate of the true effect? Explain.
- (ii) If you think that the estimate  $\beta_1$  obtained from running the regression specified above is not a good estimate of the actual/true effect, can you suggest a specification that may yield a better estimate of the true effect (of mother's divorce on her child's math grades)? Explain.
- (iii) Do you think the specification you suggested in (ii) solves the problem that is intrinsic to estimating the true value of  $\beta_1$ ? If not, why not? If yes, then why? Discuss the problem in detail and then explain.
- (iv) Now suppose you have some additional information. Some states in the United States enacted a law that made getting a divorce easier while other states did not. How would this new added information help you? Explain in detail.
- (v) Interpret the estimated value of  $\beta_1$  if the math score is graded on a scale of 0 to 100. Now, suppose one uses standardized marks ( $\text{std marks} = \frac{\text{Test score} - \text{mean}(\text{Test score})}{\text{std dev}(\text{Test score})}$ ) instead of the actual math scores in the regression specified above (std. dev being standard deviation and  $\text{std marks}$  denoting standardized marks). How would you interpret the estimated  $\beta_1$  then?

[5+5+5+5+5 = 25]