

1. Suppose five cards are drawn at random and without replacement from a well-shuffled deck of 52 playing cards (a five-card poker hand). Determine the probability of getting a full house, that is, three of a kind and two of another. The deck has 13 kinds of cards, with four of each kind. [15]
2. Find all solutions to the following system of equations.

$$\begin{aligned}x + 2y + 2z + 2u &= 0, \\2x + 4y + 5z + 7u + 2v &= 1, \\-x - 2y + 4u + 4v &= 2.\end{aligned}$$

What is the dimensionality of the set of solutions? [15]

3. Let  $A$  and  $B$  be matrices for which the matrix product is defined. Show that if the columns of  $B$  are linearly dependent, then the columns of  $AB$  are linearly dependent. [15]
4. Find the area enclosed by the curves  $y = e^x$  and  $y = 1 - e^x$  and the  $y$ -axis. [15]
5. Find

$$\lim_{x \rightarrow 0} \sin x \log x.$$

Provide a derivation. You can use standard theorems about limits. [15]

6. Let  $f$  and  $g$  be concave real-valued functions defined on the real numbers, that are not necessarily differentiable.
  - (a) Suppose  $f$  is an increasing function. Show that  $f \circ g$  given by  $f \circ g(x) = f(g(x))$  is concave.
  - (b) Give an example to show that  $f \circ g$  need not be concave if  $f$  is not increasing.

[15 + 10 = 25]