2016
Test Codes: QEA and QEB (Both are of 'Descriptive' type)
(Junior Research Fellowship in Quantitative Economics)

The candidates for Junior Research Fellowship in Quantitative Economics are required to take two descriptive-type tests - QEA in the forenoon session and QEB in the afternoon session. While the questions in QEA are on Mathematics, those in QEB are on Economics.

Syllabus for QEA
1. Permutations and combinations.
2. Theory of quadratic and cubic equations; Elementary set theory.
3. Matrix algebra, rank and inverse of matrices, linear equations, determinants, and eigenvalues.
4. Functions of one and two variables: Limits, continuity, differentiation, applications, integration of elementary functions, and definite integrals.
5. Constrained and unconstrained optimization, convexity of sets ,and concavity and convexity of functions.
6. Elements of probability theory, discrete and continuous random variables, expectation and variance, joint conditional and marginal distributions, and distribution of function of a random variable.

1. (a) If $|a|<1$ and $|b|<1$, then find if the series
$$a(a+b)+a^2(a^2+b^2)+a^3(a^3+b^3)+\ldots\ldots$$
converges or not.

(b)   Let x_1 and x_2 be the roots of the quadratic equation $x^2-3x+a=0$ and x_3 and x_4 be the roots of the quadratic equation $x^2-12x+b=0$. If x_1,x_2,x_3 and $x_4(0<x_1<x_2<x_3<x_4)$ are in G.P., then find the value of ab.

2. A bivariate probability density function is defined by
$$f_{X,Y}(x,y)=C(x+2y) \text{ if } 0<y<1 \text{ and } 0<x<2$$
$$= 0 \text{ otherwise}$$
where C is a constant.
(a) Find the value of C.
(b) Find the marginal distribution of X.
(c) Find the joint cumulative distribution function of X and Y.

3.   For the matrix $A=\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, show that there does not exist any invertible matrix Q and a diagonal matrix D such that $Q^{-1}AQ=D$.

4. Let $f(x)$ = minimum $\{x, 10 - x\}$, $x\geq 0$. For any non-negative real number t, let $x(t)$ be the (global) maxima of $f(x)$ for $x \in [0, t]$. Find the function $x(t)$.

5. Find all positive solutions of the following system of equations:

$$x_1 + x_2 = x_3^2,$$
$$x_2 + x_3 = x_4^2,$$
$$x_3 + x_4 = x_5^2,$$
$$x_4 + x_5 = x_1^2,$$
and $$x_5 + x_1 = x_2^2.$$

6. Suppose a real-valued function f over $[0, \infty)$, satisfies the following properties: (a) $f(x)$ is continuous for $x \geq 0$, (b) $f'(x)$ exists for $x > 0$, (c) $f(0) = 0$, and (d) $f'(x)$ is monotonically increasing.

Now define another real-valued function g over $(0, \infty)$, as $g(x) = \dfrac{f(x)}{x}$ for $x > 0$.

Show that $g(x)$ is a monotonically increasing function.

7. Let f be a real valued function defined on 2×2 real matrices $A = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ where $a_1 = (a_{11}, a_{12})'$ and $a_2 = (a_{21}, a_{22})'$ are any two real 2-dimensional row vectors. Further, f satisfies the following properties.

(i) f is a linear function of each row when the other row is held fixed. For example, when the second row is held fixed,

$$f\begin{pmatrix} \delta\, a_1 + a_1^* \\ a_2 \end{pmatrix} = \delta\, f\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + f\begin{pmatrix} a_1^* \\ a_2 \end{pmatrix}$$

where a_1^* is any real 2-dimensional row vector and δ is any real number.

(ii) $f\begin{pmatrix} a_1 \\ a_1 \end{pmatrix} = 0$ for all a_1.

(iii) $f\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = 1$ when $a_1 = (1,0)$ and $a_2 = (0,1)$

Show that f is the determinant of A.

8. A deck of 52 playing cards is shuffled, and the cards are turned up one at a time until the first 'ace' appears. Find which of the following two events A and B is more likely to happen.
A: The next card drawn (i.e., the card following the first 'ace') is the 'ace' of spades; and
B: The next card drawn is the '2' of clubs.

9. Let $f : \Re \to \Re$ be a twice differentiable function such that $f(\xi_i) = 0$ for $i = 1, 2, 3$ where $\xi_i's$ are distinct. Show that the second derivative of f vanishes at a point.

10. Find the value of the integral

$$\int_A x^2 e^{xy} \, dx \, dy$$

where A is the region bounded by the straight lines $y = x$, $y = 0$, and $x = 1$.