Part I

1. Let

$$f(x) = \begin{cases} \frac{\sqrt{1+2x} - \sqrt{1-2x}}{x} & \text{if } x \neq 0, \\ 2 & \text{if } x = 0. \end{cases}$$

Then $f'(0)$ is

(a) 0 (b) 1 (c) 2 (d) 3

2. A continuous random variable X has the probability density function:

$$p(x) = \begin{cases} kx^3, & \text{for } 2 \leq x \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

What is the value of k?

(a) $\frac{1}{65}$ (b) $\frac{1}{260}$ (c) $\frac{1}{19}$ (d) $\frac{1}{195}$

3. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = x^5 + x^3 + x + 1$. Then which of the following statements is correct?

(a) The function is always decreasing.

(b) The function is always increasing.

(c) The function sometimes increases and sometimes decreases.

(d) The function is not defined at certain places in $\mathbb{R}$.

4. How many solutions are there to the following system of nonlinear equations?

$$x^2 + y^2 = 41$$

$$xy = 20$$

(a) 0 (b) 1 (c) 2 (d) 4

5. What is the value of the following:

$$\lim_{x \to 0} \frac{e^x - e^{-x} - 2x}{x^3 + x^4}.$$

   (a) 0

   (b) 1/3

   (c) 1/6

   (d) The limit does not exist

6. What are the general solution to the differential equation:

$$x^2 \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + 6y = 0.$$

   (a) $y = C_1 x^2 + C_2 x^3$ where C_1 and C_2 are arbitrary constants.

   (b) $y = Cx^2$ where C is an arbitrary constant.

   (c) $y = Cx^3$ where C is an arbitrary constant.

   (d) None of the above.

7. The probability of selecting a quartz crystal from a box containing 5 quartz, 3 feldspar, and 2 mica crystals is

   (a) 1/2 (b) 1/3 (c) 1/4 (d) 1/6

8. A university is forming a committee of 5 members from a group of 8 faculty members and 6 students. The committee must include at least 2 faculty members and at least 1 student. How many different committees can be formed under these conditions?

   (a) 560 (b) 840 (c) 1400 (d) 1820

9. Consider the following matrix:

$$B = \begin{bmatrix} 3 & -1 & 0 & 2 & 1 \\ -1 & 4 & 1 & 0 & -2 \\ 0 & 1 & 5 & -1 & 3 \\ 2 & 0 & -1 & 6 & 1 \\ 1 & -2 & 3 & 1 & 7 \end{bmatrix}$$

What are the approximate values of its eigenvalues?

(a) $\lambda_1 = 8.2, \lambda_2 = 5.6, \lambda_3 = 3.8, \lambda_4 = 2.0, \lambda_5 = 0.5$

(b) $\lambda_1 = 8.2i, \lambda_2 = 5.6 + 2.3, \lambda_3 = 3.8, \lambda_4 = 2.0, \lambda_5 = 0.5$

(c) $\lambda_1 = 8.2i, \lambda_2 = 5.6 + 2.3i, \lambda_3 = 3.8 + 0.7i, \lambda_4 = 2.0, \lambda_5 = 0.5$

(d) $\lambda_1 = 8.2i, \lambda_2 = 5.6 + 2.3i, \lambda_3 = 3.8 + 0.7i, \lambda_4 = 2.0 + 9.4i,$ $\lambda_5 = 0.5$

10. What is the negation of the following statement:
"If there is no major shift in mantle convection or no dramatic change in the external planetary influences then significant changes in Earth's surface processes are unlikely."

(a) If there is no major shift in mantle convection or no dramatic change in the external planetary influences, significant changes in Earth's surface processes are still likely.

(b) If there is a minor shift in mantle convection or there is a small change in the external planetary influences, significant changes in Earth's surface processes are unlikely.

(c) A minor shift in mantle convection and a small change in the external planetary influences are necessary for a significant changes in Earth's surface processes are unlikely.

(d) Significant changes in Earth's surface processes are likely even if there is no major shift in mantle convection or no dramatic change in the external planetary influences.

Part II

1. State the equation for the model age relative to CHUR ($^{Nd}T_{CHUR}$) of a sample. Calculate the $^{Nd}T_{CHUR}$ for a rock given the following data: the present day $^{143}Nd/^{144}Nd$ for CHUR is 0.512638 and its $^{147}Sm/^{144}Nd$ is 0.1967; the present day $^{143}Nd/^{144}Nd$ for the sample is 0.513101 and its $^{147}Sm/^{144}Nd$ is 0.2365; λ_{147} is 6.539×10^{-12}.

2. Two apparent dips measured on a plane are 45°→ 070° and 50°→190°. Determine the strike and true dip of that plane.

3. The mineral assemblage of a metamorphic rock is Garnet + Biotite + Kyanite + Plagioclase + Quartz, having the following mineral compositions:

Garnet	$(Fe_{2.23}Mg_{0.47}Ca_{0.21}Mn_{0.08})Al_2Si_3O_{12}$;
Biotite	$K_{0.8}(Fe_{1.20}Mg_{1.38})Al_{1.45}Si_3O_{10}(F.OH)_2$;
Plagioclase	$(Na_{0.62}K_{0.01}Ca_{0.38})Al_{1.4}Si_{2.6}O_8$.

   Given the following equation of standard state for pure phase at equilibrium, calculate the temperature of equilibration of the assemblage at 6 kbar (R=8.314 J/mol/K).

   $$\ln\left(K_{Fe-Mg}^{Gt-Bt}\right) = -\frac{52108}{3RT} + \frac{19.51}{3R} - \frac{0.238}{3RT}P;$$

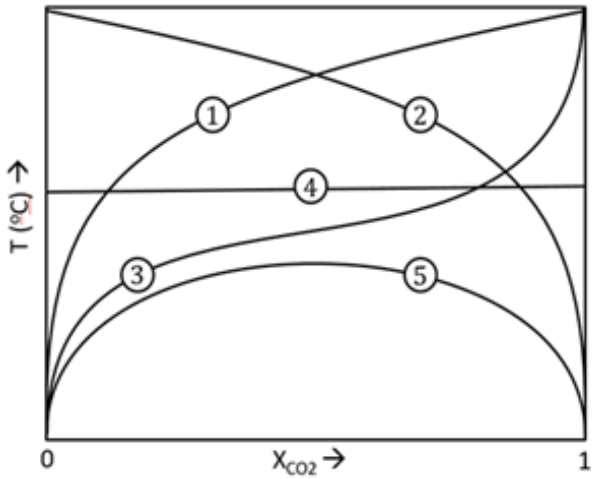
   where

   $$K_{Fe-Mg}^{Gt-Bt} = \frac{\left(\frac{Fe}{Mg}\right)^{Bt}}{\left(\frac{Fe}{Mg}\right)^{Gt}}.$$

4. The $\delta^{18}O$ values of the minerals in a vein in a metamorphic rock are: quartz=14.8, magnetite=5. Calculate the equilibrium temperature of quartz–magnetite. (A= 5.57, B=0).

5. Isotopic analyses of Mg-suite lunar crustal rocks yielded the following isotope ratios: whole rock $^{147}Sm/^{144}Nd$=0.1754 and $^{143}Nd/^{144}Nd$=0.512016; initial $^{143}Nd/^{144}Nd$=0.507039. Given that $\lambda_{147} = 6.54 \times 10^{-12}$ yr^{-1}, what is the age of the lunar crust?

6. In the given T-X diagram, identify the volatile species (i.e. CO_2 or H_2O) on the high-temperature side of each of the reactions 1, 2, 3, 4 and 5.



7. For $\frac{R_x}{R_z} < 0.732$, the cation will be too small or will rattle in its site and the structure will have to change to 6-fold coordination. Derive the lower limit of $\frac{R_x}{R_z}$ for 6-fold coordination.

8. A large area is intruded by three basaltic sills with thickness of 30, 40, and 50 m. The sill density is 2.8 Mg/m^3 and the asthenosphere density is 3.2 Mg/m^3. What is the change in the height of the surface after isostatic equilibrium has been restored?

9. Coded morphology of Seven hypothetical species (A-E) is given in the table below. If we divide these five species into two groups which of the following will be the best alternative supported by 60% characters?

	Characters					
		1	2	3	4	5
Species	A	-	-	+	-	-
	B	+	+	-	+	-
	C	-	-	-	+	-
	D	-	-	-	+	-
	E	+	+	-	-	+

10. Length and diameter of the specimens of two species are given in the table below. If we draw a frequency distribution curve of width-to-length ratio of these two species, what will be the difference between the mean values and why?

Specimen No.	Length (mm)	Diameter (mm)	Specimen No	Length (mm)	Diameter (mm)
SP 1/1	11.23	12.35	SP 2/1	12	18
SP 1/2	13.34	16.01	SP 2/2	14	15.4
SP 1/3	12.87	17.32	SP 2/3	13	20.8
SP 1/4	10.56	15.84	SP 2/4	15	21
SP 1/5	13.76	16.48	SP 2/5	11	13.2
SP 1/6	11.67	15.17	SP 2/6	12	15.60
SP 1/7	10.76	16.14	SP 2/7	15	18