Part I

1. You, along with two friends, have gone to a party. You notice that everyone at the party, except you and your friends, have shaken hands with five other people while you and your friends have shaken the hands of only one other person. You and your friends have not shaken hands with each other. The number of people at the party (including you and your friends) is at least

   (a) 6
   (b) 7
   (c) 8
   (d) 10

2. In a game, a player picks 6 distinct numbers between 1 and 9. Then, 6 balls are drawn at random (without replacement) from a total of 9 balls, numbered from 1 to 9. The probability that all the 6 numbers picked by the player is the same as the numbers on the six balls that are drawn at random is

   (a) $\frac{1}{^9C_6}$
   (b) $\left(\frac{1}{^9C_6}\right)^2$
   (c) $\frac{1}{^9P_6}$
   (d) $\left(\frac{1}{^9P_6}\right)^2$

3. In a hostel, a caterer is contracted to serve lunch every day. The caterer is supposed to serve one rice dish, one dal/sambar dish, one vegetable dish and one dessert every day. The following are the options given to the students:

	Items
Rice dish	white rice, fried rice, veg biriyani, sambar rice, tomato rice
Dal/sambar	moong dal, toor dal, masoor dal, rajma, sambar
Vegetable dish	palak paneer, brinjal curry, cauliflower with potato, cabbage curry, parval curry
Dessert	rasgolla, payesam, gulab jamoon, carrot halwa, kaju katli

If two lunches are considered different when at least one item is not the same, how many different lunches can the caterer serve?

(a) 1024

(b) 20

(c) 4

(d) 625

4. Let $M = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & \alpha \\ 2 & -\alpha & 0 \end{bmatrix}$ and $\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$, where α is non-zero and y_1, y_2, y_3 are real numbers. Let $M\mathbf{x} = \mathbf{y}$ for some vector $\mathbf{x}$. Then the value of $\mathbf{x}^T \mathbf{y}$ is

(a) α

(b) $-\alpha$

(c) 0

(d) αy_2

5. The area of the region bounded by the curves $y = x^2$ and $x = y^2$ is

(a) $\frac{1}{5}$

(b) $\frac{1}{4}$

(c) $\frac{1}{3}$

(d) $\frac{2}{3}$

6. Let
$$f(x) = \begin{cases} \frac{x^2 - 9}{x - 3} & \text{if } x < 3, \\ cx^2 + 10 & \text{if } x \geq 3. \end{cases}$$
Then the value of c that ensures $f(x)$ is continuous at $x = 3$ is

(a) $-\frac{4}{9}$

(b) $\frac{1}{9}$

(c) $\frac{2}{9}$

(d) 0

3

7. If $[\alpha]$ denotes the largest integer less than or equal to a real number α, then the value of $\int_{\alpha}^{\alpha+1} [x] \, dx$ is equal to

   (a) 1

   (b) $[\alpha]$

   (c) α

   (d) $[\alpha] + 1$

8. The $\lim_{x \to 0} \frac{(1+x)^{1/21} - 1}{(1+x)^{1/22} - 1}$

   (a) is equal to 1

   (b) is equal to $\frac{21}{22}$

   (c) is equal to $\frac{22}{21}$

   (d) does not exist

9. The range of the function $f(x) = 1 - |x|^{2/3}$ is

   (a) $(-\infty, 1]$

   (b) $(-\infty, 1)$

   (c) $(-\infty, \infty)$

   (d) $(-\infty, 0]$

10. There are two poles. The height of the first pole is 54 meters. The angles of depression of the top and foot of the second pole, measured from the the top of the first pole are 45° and 60°, respectively. Then the height of the second pole is

   (a) $54\sqrt{3}$
   (b) $\frac{54(\sqrt{3}-1)}{\sqrt{3}}$
   (c) $54(\sqrt{3}-1)$
   (d) $54\left(\frac{\sqrt{3}}{\sqrt{3}-1}\right)$

Part II

1. A large area of continent consists of 30 km of crust with density 2.8 Mg/m^3 over 90 km of material with density 3.1 Mg/m^3 . The asthenosphere density is 3.2 Mg/m^3 . This region is covered with a 1.6 km thickness of ice of density 0.9 Mg/m^3. The ice covered region is assumed to be isostatic equilibrium. Then, the ice melts. By how much will the rock surface of the continent change when the new isostatic equilibrium is re-established?

2. The stress tensor at the Earth's surface, referred to a North-East-Down coordinate system, is given by the following matrix:

$$\sigma_{ij} = \begin{bmatrix} 100 & 40 & 20 \\ 40 & 60 & 11 \\ 20 & 10 & 30 \end{bmatrix}$$

   (a) What are the normal and shear stresses on a plane that is parallel to the surface of the Earth?
   (b) What is the mean stress?

3. The results of the modal analysis for 1000 points from a sandstone sample are given in the following table. Name the sandstone.

Mineral	Count
Quartz	650
Feldspar	150
Rock Fragments	100
Clay	100

4. Chromium content of a shale on multiple determinations or replicates of a single sample is as follows –

Replicates	Cr (ppm)
1	205
2	255
3	195
4	220
5	235

Find the Sum of Squares and variance.

5. Given, $\tau = \eta(dU/dy)$. Derive the shear stress (τ) and velocity (U_y) at any depth y for a fluid that is flowing with gradient θ. Where, η is dynamic fluid viscosity, U is fluid velocity, D is depth and $\partial u/\partial y$ is shear velocity.
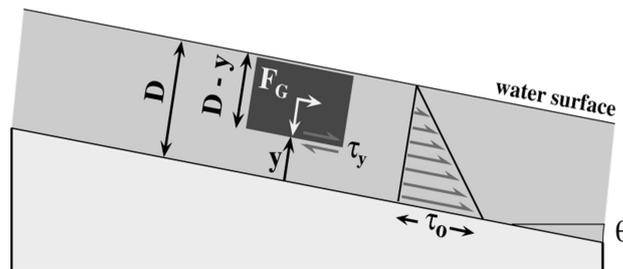
6. Coded morphology of five hypothetical species (A-E) given below.

		Characters				
		1	2	3	4	5
Species	A	-	-	-	+	-
	B	-	-	+	-	-
	C	-	+	-	+	+
	D	+	+	-	-	+
	E	+	+	-	+	-

If we divide these five species into two groups which of the following will be the best alternative supported by 60% characters.

(A) ABC & DE,

(B) ACD & BE,

(C) AD & CBE,

(D) None of the above.

7. Calculate the density of water-sediment mixture (ρ_m) with percent of sediment concentration C. Assume, density of sediment as σ_s and density of fluid as ρ_f.

8. Three wells A, B and C are located at the apices of an equilateral triangle, spaced 500m apart. A is N45°E of B. The ground level elevations of A, B and C are 60m, 42m and 38m respectively. The depth to water table in A, B and C are 12m, 6m and 2m respectively. Determine the hydraulic gradient and direction of groundwater flow.

9. The following sketch is a part of a plain table survey. The survey started from Stn. 0 and continued till Stn. 3. Which of the following equations gives the bearing (θ) of Stn. 3 from Stn. 0?
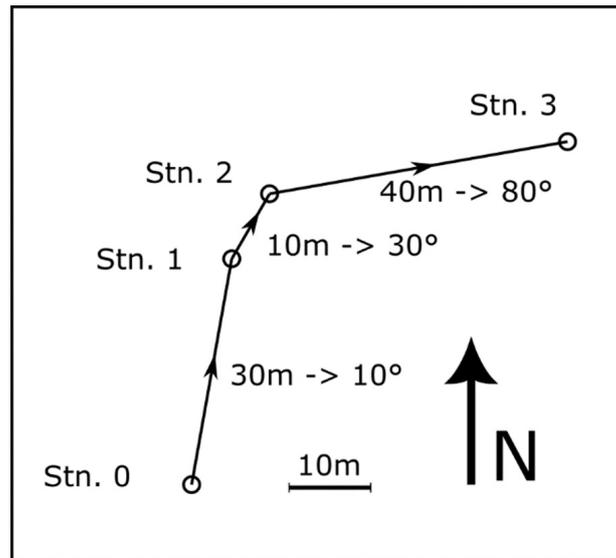
(A) $\theta = 90 - \tan^{-1}\left(\frac{30 \times \sin(80) + 10 \times \sin(60) + 40 \times \sin(10)}{30 \times \cos(80) + 10 \times \cos(60) + 40 \times \cos(10)}\right)$

(B) $\theta = \tan^{-1}\left(\frac{30 \times \sin(10) + 10 \times \sin(30) + 40 \times \sin(80)}{30 \times \cos(10) + 10 \times \cos(30) + 40 \times \cos(80)}\right)$

(C) $\theta = 90 - \tan^{-1}\left(\frac{\sin(10) + \sin(60) + \sin(80)}{\cos(10) + \cos(60) + \cos(80)}\right)$

(D) $\theta = \tan^{-1}\left(\frac{\cos(10) + \cos(30) + \cos(80)}{\sin(10) + \sin(30) + \sin(80)}\right)$

10. What is the difference between punctuated equilibrium and phyletic gradualism? Elucidate your answer by drawing two schematic graphs by putting morphology at the x-axis and time at the y-axis.