Part-I

Select the right answer from the given alternatives for each of the following questions.

10×4=40

1. The locus of the middle points of all chords of the parabola $y^2 = 8x$, which pass through the point (1,-2) is
   (a) $y^2 + 2y = 4(x - 1)$
   (b) $x^2 + 2x = 4(y - 1)$
   (c) $x^2 + 4y = 2(y - 1)$
   (d) None of these

2. The value of $\tan\left\{i \log \frac{a-ib}{a+ib}\right\}$ is
   (a) $\frac{2ab}{a^2-b^2}$
   (b) $\frac{2ab}{a^2+b^2}$
   (c) $\frac{a^2+b^2}{2ab}$
   (d) None of these

3. $\lim_{x\to\frac{\pi}{2}}(\sin x)^{\tan x}$ is equal to
   (a) 1
   (b) 0
   (c) 2
   (d) None of these

4. If $\tan y = \frac{2t}{1-t^2}$,$\sin x = \frac{2t}{1+t^2}$, then $\frac{dy}{dx}$ is
   (a) 1
   (b) 2
   (c) 3
   (d) None of these

5. If $y = a(1 - \cos\theta)$, $x = a(\theta - \sin\theta)$, y being regarded as a function of x , the function is maximum at

(a) $\theta = \pi$

(b) $\theta = 0$

(c) $\theta = \frac{\pi}{2}$

(d) None of these

6. The integral $\int_0^{\pi} \frac{xsinx}{1+cos^2 x}\, dx$ is equal to

(a) $\frac{\pi^2}{4}$

(b) $\frac{\pi^2}{2}$

(c) $\frac{\pi}{8}$

(d) None of these

7. The solution of the differential equation

$\frac{dy}{dx} + \frac{y}{x} = y^2$

is

(a) $\frac{1}{y} = cx - x\log x$

(b) $x + y = cxy$

(c) $\log xy + (x - y) = c$

(d) None of these

8. The inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}$$

is

(a) $\begin{bmatrix} \frac{1}{10} & \frac{8}{10} & \frac{1}{5} \\ \frac{21}{20} & \frac{-7}{20} & \frac{-2}{5} \\ \frac{-9}{10} & \frac{8}{10} & \frac{1}{5} \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 8 & 0 \\ 0 & 4 & -6 \\ -1 & 5 & -7 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 6 & 4 \\ 21 & -7 & -8 \\ -18 & 6 & 4 \end{bmatrix}$

(d) None of these

9. The characteristic roots and the corresponding characteristic vectors of the matrix

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 8 & -4 & 3 \end{bmatrix}.$$

are

(a) $0, (1 \quad 2 \quad 2)'$; $3, (2 \quad 1 \quad -2)'$; $15, (2 \quad 2 \quad 1)'$

(b) $1, (2 \quad 3 \quad 3)'$; $2, (1 \quad 3 \quad -3)'$; $8, (3 \quad 3 \quad 2)'$

(c) $4, (3 \quad 2 \quad 2)'$; $5, (4 \quad 2 \quad -6)'$; $10, (1 \quad 2 \quad 2)'$

(d) None of these

10. The values of θ for which the equations

$$x + y + z = 1$$
$$x + 2y + 4z = \theta$$
$$x + 4y + 10z = \theta^2$$

are consistent and the corresponding solutions are

(a) 1 , x=-1 , y=3 , z=-1 ; 2 , x=-2 , y=4, z=-1

(b) 3, x=1, y=2 , z=-2

(c) 4 , x=2 , y=3 , z= -4

(d) None of these

Part-II

(Fifteen *questions, four marks each)*

11. The ridge push (F_{rp}) and slab pull (F_{sp}) are two dominant plate driving mechanisms. These two forces can be quantified as:

a) $$F_{rp} = g\rho_m\alpha_V\left(T_m - T_0\right)\left(1 + \frac{2\rho_m\alpha_V\left(T_m - T_0\right)}{\pi\left(\rho_m - \rho_0\right)}\right)\kappa t$$

$$F_{sp} = M_e g$$

b) $$F_{rp} = g\rho_m\alpha_V\left(T_m - T_0\right)\left(1+\frac{2\rho_m\alpha_V\left(T_m - T_0\right)}{\pi\left(\rho_m - \rho_0\right)}\right)$$

$$F_{sp} = M_e g$$

c) $$F_{rp} = g\rho_m\left(T_m - T_0\right)\left(1+\frac{2\rho_m\left(T_m - T_0\right)}{\pi\left(\rho_m - \rho_0\right)}\right)\kappa t$$

$$F_{sp} = M_e g$$

d) $$F_{rp} = \rho_m\alpha_V\left(T_m - T_0\right)\left(1+\frac{2\rho_m\alpha_V\left(T_m - T_0\right)}{\pi\left(\rho_m - \rho_0\right)}\right)\kappa t$$

$$F_{sp} = M_e g$$

12. An iceberg has a density of 950.0 kg m^{-3}. What fraction of its volume is submerged in sea water with density of 1025.0 kg m^{-3}? (Hint: $\rho_1 g V_1 = \rho_2 g V_2$)

a) 92.7
b) 90.1
c) 93.0
d) 91.5

13. A tracer is diffused from a region of high concentration (C_0 at y = 0) toward a region of (at y = h) where the concentration is kept very low (by constant replacement). Assume that the diffusion coefficient D is constant. Then the concentration profile will be (Hint: Fick's second law is $d^2C/dy^2 = 0$):

a) $C = C_0(1-y/h)$
b) $C = C(y-y/h)$
c) $C = C_0(y-y/h^2)$
d) $C = C_0(h-y^2/h^2)$

14. A debris flow of 2.0 m thick moves down a slope inclined at an angle α = 5.7 degrees, equivalent to sin α = 0.1. The bulk density of the flow was found to be 2400 kg m^{-3}. The debris approximated a Bingham material with shear strength equal to 4 x 10^3 Pa, and a viscosity of 400 Pa s. The thickness of the rigid plug will be: (Hint: shear stress varies linearly with distance from surface to the flow, Y):

a) Y = 1.7m
b) Y = 17.0m
c) Y = 0.17m
d) Y = 2.0m

15. A vector $\begin{pmatrix} x \\ y \end{pmatrix}$ is transformed to $\begin{pmatrix} x' \\ y' \end{pmatrix}$ by a transformation which can be written as $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$, where $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ is the transformation matrix. If the original vector is given by $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, the transformed vector and its orientation with respect to x-axis are given by

(a) $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $\sin^{-1}(1/\sqrt{3})$

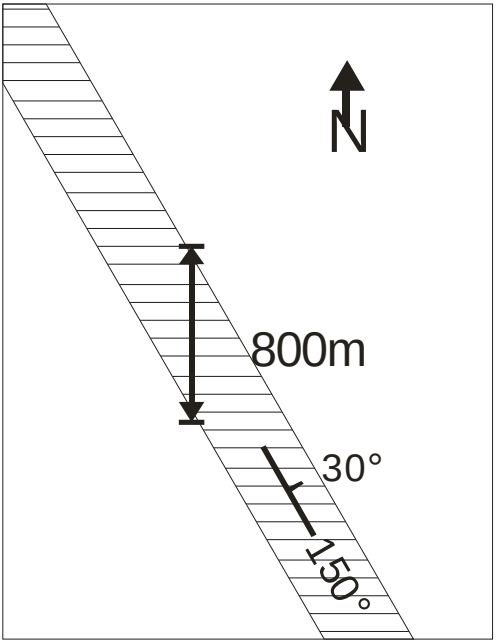
(b) $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $\sin^{-1}(1/\sqrt{2})$

(c) $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\sin^{-1}(1/\sqrt{5})$

(d) $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $\sin^{-1}(1/\sqrt{5})$.

16. Slickensides on a fault plane dipping 60° toward East show a pitch of 30° N. A horizontal coal seam displaced by this oblique slip fault shows a vertical throw of 50 metres, with hanging wall going down relative to footwall. The correct amount of strike slip component in metres is given by

(a) 100/3
(b) 100/√3
(c) 100√3
(d) 100/2.



17. The true thickness of the bed (shown by hatch symbol) is:

a) 200m
b) 300m
c) 800m
d) 1600m

18. 500 chert pebbles measured on a beach have mean sphericity of 0.71, standard deviation is 0.08. How many pebbles are expected to have sphericity values greater than 0.79, assuming the distribution is normal?

a) 92.
b) 79.
c) 40.
d) 57.

19. A cladogram is based on the hierarchical progression of shared characters and the character states can be:

a) Both numeric and descriptive (non numeric).
b) Only numeric.

c) 90% numeric.
d) 80% numeric.

20. Length of five specimens of a temnospondyl amphibian measured in mm are; 205, 255, 220, 195, 235. The 'Variance (s)' of the length data will be:

a) 23.88.
b) 1110.
c) 570.
d) 2280
e)

21. If the fluid potential is given by $\varphi = x^2+xy+yz$ then at the point (2, 1, 4) the coordinates of the unit vector pointing in the direction of maximum rate of change of the potential will be:

a) $\mathbf{r} = 0.635\mathbf{i}+0.762\mathbf{j}+0.127\mathbf{k}$
b) $\mathbf{r} = 0.65\mathbf{i}+0.72\mathbf{j}+0.17\mathbf{k}$
c) $\mathbf{r} = 0.0635\mathbf{i}+0.0762\mathbf{j}+0.0127\mathbf{k}$
d) $\mathbf{r} = 6.35\mathbf{i}+7.62\mathbf{j}+1.27\mathbf{k}$

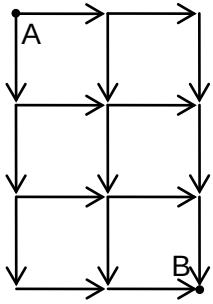
22. In a volcanic eruption from a cone which rises 300 m above the surrounding (flat) countryside, large bombs are observed to be thrown a maximum distance of 3000 m. neglecting air resistance, and assuming that the initial angle of ejection was 45 degrees, the speed at which they will hit the ground will be:

a) 180.6 m s^{-1}
b) 18.06 m s^{-1}
c) 1.806 m $s^{-1}\mathbf{r}$
d) 0.1806 m s^{-1}

23. If the left and the right banks of a river could be represented by the following two equations, respectively:
$y = 50 \sin(x) + 150$ and $y = -30 \cos(x) + 60$

Then the maximum width of the river along y is:

a) 170
b) 140
c) 120
d) 80



24. The channel segments in a braided river system are shown. Water in the channels can move only in the directions indicated by the arrow heads (i.e., either to the east or to the south). If the total number of east-west segment is ***e*** and north-south segment is ***s*** then the number of total number of paths through which water can flow from A to B is:

a) $\frac{(e+s)!}{e!\,s!}$
b) $\frac{e!+s!}{e!\,s!}$
c) $e! + s!$
d) $e!\,s!$

25. If a one centimeter cube block of sediment is internally made up of perfectly spherical grains of 10mm diameter then the maximum porosity is:

a) Less than 35%
b) Less than 25%
c) Greater than 45%
d) Greater than 55%