Part-I

Select the right answer from the given alternatives for each of the following questions.

10×4=40

1. The locus of the middle points of all chords of the parabola $y^2 = 8x$, which pass through the point (1,-2) is
    - (a) $y^2 + 2y = 4(x - 1)$
    - (b) $x^2 + 2x = 4(y - 1)$
    - (c) $x^2 + 4y = 2(y - 1)$
    - (d) None of these

2. The value of $\tan\left\{ i \log \frac{a-ib}{a+ib} \right\}$ is
    - (a) $\frac{2ab}{a^2 - b^2}$
    - (b) $\frac{2ab}{a^2 + b^2}$
    - (c) $\frac{a^2 + b^2}{2ab}$
    - (d) None of these

3. $\lim\limits_{x \to \frac{\pi}{2}} (\sin x)^{\tan x}$ is equal to
    - (a) 1
    - (b) 0
    - (c) 2
    - (d) None of these

4. If $\tan y = \frac{2t}{1-t^2}$, $\sin x = \frac{2t}{1+t^2}$, then $\frac{dy}{dx}$ is
    - (a) 1
    - (b) 2
    - (c) 3
    - (d) None of these

5. If $y = a(1 - \cos\theta)$, $x = a(\theta - \sin\theta)$, y being regarded as a function of x , the function is maximum at
    - (a) $\theta = \pi$
    - (b) $\theta = 0$
    - (c) $\theta = \frac{\pi}{2}$
    - (d) None of these

6. The integral $\int_0^\pi \frac{x\sin x}{1+\cos^2 x} \, dx$ is equal to
    - (a) $\frac{\pi^2}{4}$
    - (b) $\frac{\pi^2}{2}$

(c) $\dfrac{\pi}{8}$

(d) None of these

7. The solution of the differential equation
$$\dfrac{dy}{dx} + \dfrac{y}{x} = y^2$$
is

(a) $\dfrac{1}{y} = cx - x \log x$

(b) $x + y = cxy$

(c) $\log xy + (x - y) = c$

(d) None of these

8. The inverse of the matrix
$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}$$
is

(a) $\begin{bmatrix} \dfrac{1}{10} & \dfrac{8}{10} & \dfrac{1}{5} \\ \dfrac{21}{20} & \dfrac{-7}{20} & \dfrac{-2}{5} \\ \dfrac{-9}{10} & \dfrac{8}{10} & \dfrac{1}{5} \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 8 & 0 \\ 0 & 4 & -6 \\ -1 & 5 & -7 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 6 & 4 \\ 21 & -7 & -8 \\ -18 & 6 & 4 \end{bmatrix}$

(d) None of these

9. The characteristic roots and the corresponding characteristic vectors of the matrix
$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 8 & -4 & 3 \end{bmatrix}.$$
are

(a) $0,(1 \quad 2 \quad 2)';\ 3\ ,\ (2 \quad 1 \quad -2)'\ ;\ 15\ ,(2 \quad 2 \quad 1)'$

(b) $1,\ (2 \quad 3 \quad 3)';\ 2\ ,\ (1 \quad 3 \quad -3)'\ ;\ 8\ ,\ (3 \quad 3 \quad 2)'$

(c) $4\ ,\ (3 \quad 2 \quad 2)'\ ;\ 5\ ,(4 \quad 2 \quad -6)';\ 10\ ,\ (1 \quad 2 \quad 2)'$

(d) None of these

10. The values of θ for which the equations
$$x + y + z = 1$$
$$x + 2y + 4z = \theta$$
$$x + 4y + 10z = \theta^2$$

are consistent and the corresponding solutions are

(a) 1 , x=-1 , y=3 , z=-1 ; 2 , x=-2 , y=4, z=-1

(b) 3, x=1, y=2 , z=-2

(c) 4 , x=2 , y=3 , z= -4

(d) None of these

Part-II

(Ten questions, six marks each)

11. Consider the layered formation shown in the figure below. Each layer (1, 2, 3, 4, ..., n) is homogeneous and isotropic with hydraulic conductivity values K_1, K_2, K_3, K_4, ..., K_n, but the system as a whole will acts like a single homogeneous, anisotropic layer. Now consider flow perpendicular and parallel to the layering. Assume flux Q entering and leaving the system is constant throughout (Q = KA(dh/dl)).
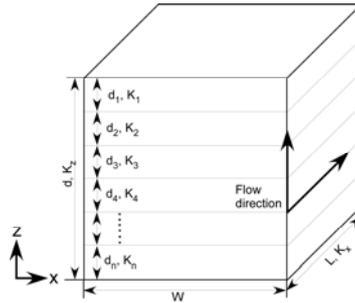
Then the vertical and horizontal hydraulic conductivity (K_z and K_x respectively) of the layered formation will be

i) $K_z = \dfrac{1}{\sum\limits_{i=1}^{n} \dfrac{d_i}{K_i}}$ and $K_x = \dfrac{\sum\limits_{i=1}^{n} d_i K_i}{d_i}$

ii) $K_z = \sum\limits_{i=1}^{n} \dfrac{d_i}{K_i}$ and $K_x = \sum\limits_{i=1}^{n} d_i K_i$

iii) $K_z = \dfrac{d}{\sum\limits_{i=1}^{n} \dfrac{1}{K_i}}$ and $K_x = \dfrac{\sum\limits_{i=1}^{n} K_i}{d}$

iv) $K_z = \dfrac{d}{\sum\limits_{i=1}^{n} \dfrac{d_i}{K_i}}$ and $K_x = \dfrac{\sum\limits_{i=1}^{n} d_i K_i}{d}$

12. A coal seam is encountered at boreholes A, B and C at depths of 100m, 200m and 200m respectively. Boreholes B and C are located 100m to the East and 100m to the North respectively of A. The strike and dip of the coal seam is best given by:

   i)   130, 40° NE
   ii) 135, 45° NE
   iii) 120, 45° NE
   iv) 135, 55° NE.

13. Following table provides the chemical compositions of ten analyzed mineral grains as oxide weight percent with an analytical error of ± 2 %.

Analysis #	Na_2O	MgO	SiO_2	Al_2O_3	K_2O	CaO	TiO_2	Cr_2O3	MnO	FeO	P_2O_5	Total
1	0.21	0.03	63.64	18.61	16.54	0.02	0.09	0.07	0.04	0.08	0	99.32
2	0.18	0.02	63.4	18.32	17.48	0	0	0	0	0.07	0	99.46
3	0.09	0	64.36	18.6	17.41	0	0.14	0	0	0	0	100.61
4	0.12	0.02	63.71	18.18	16.96	0	0.01	0.04	0	0.26	0	99.31
5	0.07	0.05	0.31	0	0.06	56.63	0	0	0	0.79	42.39	100.3
6	0.03	0.07	0.27	0.02	0.04	55.87	0	0	0.15	0.71	42.23	99.37
7	0.33	0.08	62.56	19.44	16.61	0	0.07	0	0	0.27	0	99.37
8	0.18	0	64.5	18.58	16.85	0	0.06	0.01	0.05	0.1	0.02	100.36
9	0.18	0.16	63.64	18.66	16.93	0	0.06	0	0	0.44	0	100.08
10	0.03	0	99.49	0.17	0.02	0	0	0	0.07	0.04	0	99.81

One of the mineral species is feldspar. Which of the following equations would give the best estimate of average value of K_2O in feldspar?

i) $\dfrac{\sum\limits_{i=1}^{10}(K_2O)_i}{10}$

ii) $\dfrac{\sum\limits_{i=1}^{4}(K_2O)_i}{4}+\dfrac{\sum\limits_{i=7}^{9}(K_2O)_i}{3}$

iii) $\dfrac{\left(\dfrac{\sum\limits_{i=1}^{4}(K_2O)_i}{4}+\dfrac{\sum\limits_{i=7}^{9}(K_2O)_i}{3}\right)}{2}$

iv) $\dfrac{\sum\limits_{i=7}^{9}(K_2O)_i}{3}$

14. A systematist is exploring a planet in another solar system and discovers the following four different species, A, B, C and D.
The character matrix generated is as follows with '0' as primitive and '1' as derived character states

Character	A	B	C	D
1. "Feet" present	0	0	0	0
2. Spines present	0	1	0	0
3. Eyes on stalks	0	1	1	1
4. "Tail" present	0	0	1	1
5. Antennae present	0	0	0	1

Cladogram drawn on this would group

i)   C and D first
ii)   A and C first
iii)   B and C first
iv)   A and D first

15. Carefully study the figure given below. If the densities of the mantle (ρ_m), crust (ρ_c) and water (ρ_w) are assumed to be 3,300 kg m^{-3}, 2,750 kg m^{-3} and 1000 kg m^{-3} respectively then the correct relationship between b_1, h_1 and b_2, h_w is:
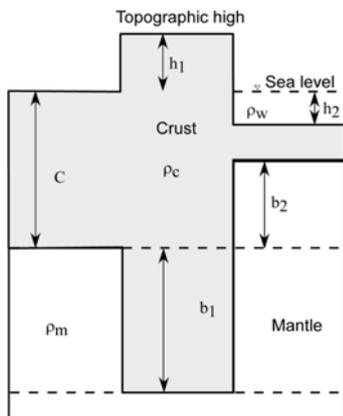
i) $b_1 \approx 5h_1$ and, $b_2 \approx 3.2h_w$
ii) $b_1 \approx 2.5h_1$ and, $b_2 \approx 2.3h_w$
iii) $b_1 \approx 0.5h_1$ and, $b_2 \approx 0.32h_w$
iv) $b_1 \approx 50h_1$ and, $b_2 \approx 32h_w$

16. A person had measured the length of long axis of 20 pebbles and had found the mean length to be 2.4cm. He tabulated his observations as follows:

Length range in cm	Number of observations
0.0-1.0	2
1.0-2.0	x
2.0-3.0	8
3.0-4.0	y
4.0-5.0	2

The number of observations noted for the two class means (i.e., x and y) were smudged beyond legibility as his note book fell into the river. If the table is reconstructed then these missing values will be:

i)   x=5 and y=3
ii)   x=4 and y=4
iii)   x=3 and y=5
iv)   x=6 and y=2

17. What type of equation best represents the relationship between the variables plotted along X and Y axes in the plot given below?
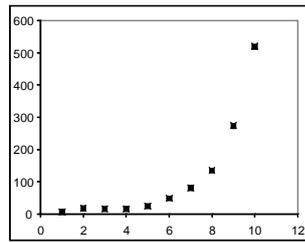
    i)   Linear
    ii)   Exponential
    iii)   Polynomial (order 2)
    iv)   No relationship

18. A 2D circular marker on a rock body has been deformed to become an ellipse having 32 cm and 8 cm as major and minor axes. Assuming volume constant homogeneous rock deformation the radius of the original undeformed marker is.

    i)   10 cm
    ii)   25 cm
    iii)   16 cm
    iv)   2.5 cm

19. A bed striking N-S and dipping at an angle 45 degrees to the west has been displaced by a vertical fault striking E-W. If on a horizontal outcrop the amount of displacement of the bed is 500 meters, assuming pure dip-slip movement, the amount of slip is:

    i)   200m
    ii)   300m
    iii)   400m
    iv)   500m

20. In a cross-bedded sandstone the axes of trough cross strata are oriented towards 355°, 350°, 005°, 285°, 075°, 010°, and 0°. The mean paleocurrent direction is approximately towards:

    i) 90°
    ii) 180°
    iii) 0°
    iv) 235°