1. Consider an infinite G.P. If the sum of its terms is 8 and the sum of the squares of its terms is 32, then the common ratio of G.P. is

   (A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$ (D) $\frac{1}{8}$

2. The number of triangles that can be formed using 9 points lying on two parallel lines, where four of those points are on one line and the rest are on the other line, is

   (A) 60 (B) 70 (C) 80 (D) 84

3. Six people $P_1, P_2, \cdots, P_6$ are to be seated in a row. The number of possible seating arrangements such that P_1 and P_6 are not next to each other is

   (A) 360 (B) 420 (C) 480 (D) 600

4. The absolute value of k which satisfies the relation

   $$\sqrt{15^2\sqrt{15^2\sqrt{15^2\cdots}}} - \sqrt{12^2\sqrt{12^2\sqrt{12^2\cdots}}} = \sqrt{k^2\sqrt{k^2\sqrt{k^2\cdots}}}$$

   is

   (A) 3 (B) $3\sqrt{3}$ (C) 9 (D) 27

5. The minimum value of $f(z) = |z+2i| + |z-2i|$, where z is a complex number, is

   (A) 2 (B) 3 (C) $2\sqrt{2}$ (D) 4

6. If α is a real positive number for which $x^2 - 3\alpha x + 1$ has real roots and $x^2 - 5\alpha x + 4$ has complex roots, then the set of all values of α is

   (A) $(\frac{2}{3}, \frac{4}{5})$ (B) $[\frac{2}{3}, \frac{4}{5})$ (C) $(\frac{2}{3}, \frac{4}{5}]$ (D) $[\frac{2}{3}, \frac{4}{5}]$

7. If the term free of x in the expansion of $\left(\sqrt{x} - \frac{\beta}{x^2}\right)^{15}$ is 455, then the value of β is

(A) -1 (B) 1 (C) -5 (D) 5

8. If the system of linear equations

$$\begin{aligned} x + y + z &= \alpha \\ x + 2y + 3z &= \beta \\ \alpha x + \beta y + 5z &= 5 \end{aligned}$$

has infinitely many solutions, then the values of α and β, respectively, are

(A) -1 and 2 (B) 3 and 4 (C) 1 and 3 (D) 1 and 2

9. Let I be the identity matrix of order 3 and A be a 3×3 matrix. If $A\begin{pmatrix}1\\1\\1\end{pmatrix} = 3\begin{pmatrix}1\\1\\1\end{pmatrix}$, $A\begin{pmatrix}0\\1\\1\end{pmatrix} = 2\begin{pmatrix}0\\1\\1\end{pmatrix}$ and $A\begin{pmatrix}1\\2\\3\end{pmatrix} = \begin{pmatrix}3\\5\\9\end{pmatrix}$, then $(A - 4I)\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$ has

(A) no solution (B) a unique solution

(C) exactly two solutions (D) infinitely many solutions

10. Let $M = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & -1 & -1 \end{bmatrix}$ and A be the adjoint of M. Then the determinant $|A\,M^3 - 4M|$ is equal to

    (A) 32 (B) -32 (C) 2 (D) 8

11. Let

$$M = \begin{bmatrix} \cos x & 2\sin x & \sin x \\ x & x & x \\ 1 & 2x & x \end{bmatrix}.$$

    Then $\lim_{x\to 0} \frac{|M|}{x^2}$ is equal to

    (A) -1 (B) 0 (C) 1 (D) 2

12. Let x_i, $i \geq 0$, be the sequence satisfying the relation

$$x_{i+4} = x_{i+2} + 2x_{i+1} + x_i$$

    with $x_0 = 0, x_1 = 1, x_2 = 1$ and $x_3 = 2$. Then the rank of the matrix

$$C = \begin{bmatrix} x_0 & x_1 & \cdots & x_9 \\ x_{10} & x_{11} & \cdots & x_{19} \\ \vdots & \vdots & \ddots & \vdots \\ x_{90} & x_{91} & \cdots & x_{99} \end{bmatrix}$$

    is equal to

    (A) 2 (B) 4 (C) 5 (D) 10

13. The values of x for which $(x+3-2\min\{x,3\})(x+1)>0$ are

(A) $-1 \leq x \leq 1$ (B) $-3 \leq x \leq 3$
(C) $x > -3$ (D) $x > -1$

14. The maximum possible value of α for which the three lines $x+2y-1=0,\ 3x-y+1=0$ and $\alpha x+y=0$ do not form a triangle is

(A) 3 (B) $\frac{5}{4}$ (C) 4 (D) $\frac{1}{2}$

15. The number of values of k such that the line $x+y=k$ touches the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ is

(A) exactly 1 (B) exactly 2

(C) infinitely many (D) exactly 0

16. If the sum of the squares of the perpendicular distances of a point (x,y) from the lines $ax+by+c=0$ and $bx-ay+d=0$ is constant, then the locus of the point is

(A) a straight line (B) a parabola

(C) a circle (D) an ellipse

17. The value of

$$\lim_{x\to 0}\frac{1}{x}\int_0^x (1-\tan 2t)^{\frac{1}{t}}\ dt$$

is equal to

(A) e^{-2} (B) e^{-1} (C) e (D) e^2

18. Let $\{a_n\}$ and $\{b_n\}$ be sequences in $\mathbb{R}$ such that $a_1 = \frac{1}{2}$, $b_1 = 1$, $a_n = (a_{n-1}b_{n-1})^{\frac{1}{2}}$ and $\frac{1}{b_n} = \frac{1}{2}(\frac{1}{a_n} + \frac{1}{b_{n-1}})$. Which one of the following is correct?

    (A) Both $\{a_n\}$ and $\{b_n\}$ converge to the same limit.
    (B) Both $\{a_n\}$ and $\{b_n\}$ converge, but to different limits.
    (C) One of the sequence converges, but the other is divergent.
    (D) Both $\{a_n\}$ and $\{b_n\}$ are divergent.

19. The value of the integral $\int_{-4}^{4} |x-3|\, dx$ is

    (A) 8 (B) 12 (C) 16 (D) 25

20. For $x > 0$, the area of the region enclosed by the curves $x = y$, $xy = 1$ and $x = 4y$ is

    (A) $\log_e 2$ sq. units (B) $1 + \log_e 2$ sq. units
    (C) $\frac{1}{4} + \log_e 2$ sq. units (D) $-\frac{1}{4} + \log_e 2$ sq. units

21. The value of $\displaystyle\sum_{n=0}^{\infty} \frac{n^2+5n+5}{(n+3)!}$ equals to

    (A) $\frac{1}{2}$ (B) $\frac{5}{3}$ (C) $\frac{3}{2}$ (D) $\frac{5}{6}$

22. The value of

$$\lim_{n\to\infty} \left[\frac{n+1}{n^2+1} + \frac{n+2}{n^2+4} + \frac{n+3}{n^2+9} + \cdots + \frac{n+n}{n^2+n^2} \right]$$

equals to

    (A) $\dfrac{\pi}{4} + \dfrac{1}{2}\log 2$ (B) $\dfrac{\pi}{4} + \log 2$
    (C) $\log 2$ (D) $\dfrac{1}{2}\log 2$

23. Let $\mathbb{R}$ be the set of real numbers and $Y = \{x \in \mathbb{R} \mid \log_e(x) < 0\}$.

Which option among the following is true?

(A) Y is an open set in $\mathbb{R}$.
(B) Y is a closed set in $\mathbb{R}$.
(C) Y is an empty set.
(D) Y is neither open nor closed in $\mathbb{R}$.

24. Consider the function $K(x) = \min\{(x-2)^2, x^3\}$, where $x \in \mathbb{R}$, and $\mathbb{R}$ denotes the set of real numbers. Consider the following statements: (i) K is continuous on $\mathbb{R}$; (ii) K is differentiable in $\mathbb{R}$; and (iii) K attains a local maximum. Then which option among the following is correct?

(A) Only (i) is true.
(B) Both (i) and (ii) are true.
(C) Only (iii) is true.
(D) Both (i) and (iii) are true.

25. A farmer has 25 coconut trees in his farm. Each tree produces 150 coconuts in a year. For every additional tree planted in the farm, the output of each tree (including the pre-existing ones) drops by 5 coconuts. How many tree(s) should be added to the farm in order to maximize the total production of coconuts?

(A) 1 (B) 2 (C) 6 (D) 10

26. Let A and B be two events such that $P(\overline{A \cup B}) = \frac{1}{6}$, $P(A \cap B) = \frac{1}{4}$ and $P(\overline{A}) = \frac{1}{4}$. Then the events A and B are

(A) equally likely but not independent.
(B) independent but not equally likely.
(C) independent and equally likely.
(D) neither equally likely nor independent.

27. Consider a lot of N items where each item has an equal probability p of being defective. Suppose the acceptance of an entire lot of N items is decided by a random sample of size n drawn from the lot. Let d be the number of defective items in a sample. If $d \leq c$ the entire lot is accepted, where c is a predetermined number. Let D denote the number of defective items in the lot. Then the probability of acceptance of the lot when $c = 0$ is

(A) $\dfrac{D}{N}$ (B) $\dfrac{\binom{D}{d}\binom{N-D}{n-d}}{\binom{N}{n}}$ (C) $\dfrac{\binom{D}{d}}{\binom{N}{n}}$ (D) $\dfrac{\binom{N-D}{n}}{\binom{N}{n}}$

28. Suppose that a four-sided fair die, with its faces numbered 1, 2, 3, 4, is rolled once. If the result is either 1 or 2, then the die is rolled second time, otherwise not. Let E be the event that the sum of the outcome(s) of the rolls is at least 4. Then the probability of E is

(A) $\dfrac{9}{16}$ (B) $\dfrac{10}{16}$ (C) $\dfrac{11}{16}$ (D) $\dfrac{13}{16}$

29. There are two baskets B_1 and B_2, where B_1 contains 20 red, 30 green and 40 blue balls whereas B_2 contains 30 red, 20 green and 20 blue balls. The probabilities of selecting the baskets B_1 and B_2 are $\frac{9}{16}$ and $\frac{7}{16}$, respectively. A basket is selected at random, and then a ball is drawn at random from it. Given that the ball drawn is green, the probability that it was drawn from the basket B_1 is

(A) $\frac{1}{3}$ (B) $\frac{1}{16}$ (C) $\frac{3}{5}$ (D) $\frac{5}{16}$

30. Suppose that f is a real valued differentiable function such that $f(20) = 0$ and $400 \leq f'(x) \leq 405$ for all $x \in \mathbb{R}$. Then which one of the following is not a possible value of $f(25)$?

(A) 2015 (B) 2020 (C) 2025 (D) 2030