1. If a, b and c are real numbers such that $a > b > c > 0$ then the value of $\lim_{n\to\infty}(a^n + b^n - c^n)^{\frac{1}{n}}$ is equal to

   (A) c (B) $2c$ (C) a (D) $2a$

2. The value of the integral $\int_2^{2024} (x - \lfloor x \rfloor)^2 \, dx$ (where $\lfloor x \rfloor$ is the greatest integer less than or equal to x) is

   (A) 674 (B) 1012 (C) $\frac{2024}{3}$ (D) $\frac{2023}{3}$

3. The area bounded by the curves $y = x^2$ and $y = 2 - x^2$ is equal to

   (A) $\frac{2}{3}$ (B) $\frac{16}{3}$ (C) $\frac{8}{3}$ (D) $\frac{5}{3}$

4. The sum of first n terms of a geometric progression (GP) is p and the sum of first $2n$ terms is $3p$. Then the sum of first $3n$ terms is

   (A) $5p$ (B) $7p$ (C) $9p$ (D) $6p$

5. The mean and variance of height of male students in a university are 150 cm and 25 cm^2 respectively. The height of these students follow Normal distribution. Then the probability that height of a randomly chosen male student from this university to lie between 135 cm and 165 cm is

   (A) 0.9973 (B) 0.8889 (C) 0.9999 (D) 0.9500

6. A point P on the line $3x + 5y = 15$, which is equidistant from both the coordinate axes, lies in

   (A) 1^{st} quadrant only
   (B) 1^{st} or 2^{nd} quadrant
   (C) 1^{st} or 3^{rd} quadrant
   (D) Any quadrant

7. The events A and B are such that $P(A \cup B) = \frac{2}{3}$ and $P(A \cap B^c) = \frac{1}{3}$. Then $P(B)$ is

   (A) 1 (B) $\frac{1}{9}$ (C) $\frac{1}{3}$ (D) 0

8. Consider the system of equations in three variables x, y and z:

$$\begin{aligned} x + y &= \alpha \\ x + z &= \beta \\ y + z &= \gamma \end{aligned}$$

   Which of the following is correct?

   (A) For no values of α, β, γ, the system has a solution
   (B) The system always has a unique solution
   (C) The system may have infinite solutions
   (D) The system has a solution if and only if $\alpha = \beta = \gamma = 0$

9. Suppose $I_{2\times 2}$ is the Identity matrix. Let $B_{2\times 2}$ be any other matrix. Then which of the following statements is true?

   (A) If $B_{2\times 2}$ is not invertible, then $I_{2\times 2} + B_{2\times 2}$ is not invertible
   (B) Even if $B_{2\times 2}$ is not invertible, still $I_{2\times 2} + B_{2\times 2}$ may be invertible
   (C) If $B_{2\times 2}$ is invertible, then $I_{2\times 2} + B_{2\times 2}$ is also invertible
   (D) None of the above

10. Let $i = \sqrt{-1}$ and let k be the smallest positive integer such that the complex number $(\sqrt{3}+i)^{k+2}$ is real. If $p = 506k$, then the value of $\displaystyle\sum_{n=1}^{p} \frac{1}{i^n} + \prod_{n=1}^{p} i^n$ is

(A) -1 (B) 0 (C) 1 (D) -2

11. If $a+b+c = 1$ where a, b and c are positive real numbers, then $a^2+b^2+c^2$ is

(A) Greater than equal to 1 (B) Equal to 1
(C) Greater than equal to $\frac{1}{3}$ (D) Equal to $\frac{1}{3}$

12. There are 30 balls in a box, 10 of which are red and 20 are blue. We pick up two balls, at a time, at random from the box. What is the probability that both are of the same color?

(A) $\frac{235}{435}$ (B) $\frac{45}{435}$ (C) $\frac{190}{435}$ (D) $\frac{45}{235}$

13. The interval in which the function $f(x) = x^3 - 3x + 1$ is decreasing in x is

(A) $(-\infty, 1)$ (B) $(1, \infty)$ (C) $(-\infty, \infty)$ (D) $(-1, 1)$

14. The equation $2x^2 + 3y^2 + 4xy + 5x + 6y + 7 = 0$ represents a

(A) a circle (B) a parabola
(C) an ellipse (D) a pair of straight lines

15. The mean monthly salary paid to all employees in a certain company was Rupees 500. The mean monthly salaries paid to male and female employees were Rupees 520 and Rupees 420, respectively. Then the percentage of male and female employees in the company are

(A) 70 and 30 (B) 60 and 40

(C) 80 and 20 (D) 90 and 10

16. Consider $A_{3\times 3}$ matrix whose first row is $1, 0, 0$, the second row is $0, 0, 1$ and the third row is $0, 1, 0$. Then

(A) The matrix A is the inverse of matrix A

(B) The matrix A does not have an inverse

(C) The matrix A is the transpose of matrix A

(D) Both (A) and (C) are true

17. If the coefficient of x^7 of the $(ax^2 + \frac{1}{bx})^8$ is equal to the coefficient of x^{-7} of the $(ax - \frac{1}{bx^2})^8$ then

(A) $a^2b^2 = -1$ (B) $ab = 2$

(C) $a^2b = 1$ (D) $ab^2 = 2$

18. Consider a straight line (denoted by l) whose x and y intercepts are 4 and 3 respectively. Then perpendicular distance from the origin $(0, 0)$ to line l is

(A) $\frac{4}{3}$ (B) $\frac{3}{4}$ (C) $\frac{12}{5}$ (D) $\frac{20}{3}$

19. The minimum value of the function f, defined by $f(x) = \int_0^{\pi} \cos t \cos(x-t)dt$, for $0 \leq x \leq 2\pi$, is equal to

(A) $-\frac{\pi}{2}$ (B) 0 (C) $\frac{\pi}{2}$ (D) π

20. There are 10 members in a club who are eligible to form a governing committee that comprises of one president, one vice-president and three other members. The number of ways in which this can be done is

(A) 720 (B) 5000 (C) 5040 (D) 30240

21. Consider a magnetic compass. We marked a line connecting the centre of the compass and a point on the perimeter. Let the random variable X be the angle between the line and the magnetic needle which points toward the north pole. Then the value of $P(X \leq x)$ is

(A) $\frac{x}{2\pi}$ (B) $1 - \frac{x}{2\pi}$ (C) $\frac{x}{\pi}$ (D) $1 - \frac{x}{\pi}$

22. The limit of $\frac{x^2}{x + e^x}$, as $x \to \infty$, is

(A) 0 (B) 1 (C) 0.5 (D) ∞

23. The slope of a function $y = f(x)$ is given by $2x$. Suppose the y intercept of the function is 9. What is the form of $f(x)$?

(A) $x^2 + 9$ (B) $x^2 - 9$ (C) $2x^2 + 9$ (D) $2x^2 - 9$

24. Let $M = ((m_{ij}))$ be a 5×5 matrix with $m_{i,i+1} = 1$ for $1 \leq i \leq 4$ and $m_{5,1} = 1$ and all other elements of the matrix are zero. Then which of the following statements is false?

(A) $|M| = 1$

(B) $\text{trace}(M) = 0$

(C) $M^{-1} = M$

(D) $M^5 = I$

25. The equation of the line through the intersection of the lines $x + y + 1 = 0$ and $x - y + 1 = 0$ and perpendicular to the line $2x + 3y + 4 = 0$ is

(A) $3x + 2y + 3 = 0$ (B) $3x - 2y + 3 = 0$

(C) $3x - 2y - 3 = 0$ (D) $3x + 2y - 3 = 0$

26. Box B_1 contains 3 white and 5 red balls, and box B_2 contains 6 white and 4 red balls. One box is selected at random. The box B_1 is selected with probability $\frac{2}{3}$ and B_2 with probability $\frac{1}{3}$. A ball is then drawn at random from the selected box. The probability that it is a red ball is

(A) $\frac{33}{60}$ (B) $\frac{49}{60}$ (C) $\frac{25}{60}$ (D) $\frac{41}{60}$

27. Consider a matrix, $A_{n\times n}$, where $n \geq 1$ and n is a natural number. A matrix is called a zero matrix if and only if all the elements of the matrix are 0s. Otherwise, it is called as a non-zero matrix. Then, which of the following statements is correct?

(A) For all n, all non-zero matrices have rank n

(B) There exists n for which all non-zero matrices have rank n

(C) There exists n for which all non-zero matrices have rank less than n

(D) None of the above

28. Suppose E and F are two events with $P(E) = \frac{1}{5}$ and $P(F) = \frac{1}{3}$. Which of the following statements is correct?

(A) If E and F are independent, then the probability that at least one of them occurs is $\frac{8}{15}$

(B) If E and F are mutually exclusive, then the probability that at least one of them occurs is $\frac{7}{15}$

(C) If E and F are independent, then the probability that E occurs but F does not occur is $\frac{2}{15}$

(D) If E and F are independent, then the probability that neither event occurs is $\frac{12}{15}$

29. Let $a_n = \frac{(-1)^{n-1}}{\sqrt{(n(n+1)(n+2))}}$ and $b_n = \frac{(n!)^2}{(2n)!}5^n$, where $n \in N$. Then which one of the following statements is correct?

(A) Both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converges

(B) $\sum_{n=1}^{\infty} a_n$ converges and $\sum_{n=1}^{\infty} b_n$ diverges

(C) $\sum_{n=1}^{\infty} a_n$ diverges and $\sum_{n=1}^{\infty} b_n$ converges

(D) Both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ diverges

30. The value of

$$\lim_{n\to\infty}\left(\frac{1}{\sqrt{n}\sqrt{n+1}}+\frac{1}{\sqrt{n}\sqrt{n+2}}+\ldots+\frac{1}{\sqrt{n}\sqrt{n+n}}\right) \text{ is}$$

(A) $2(\sqrt{2}-2)$ (B) $2\sqrt{2}-1$ (C) $2(\sqrt{2}-1)$ (D) $2\sqrt{2}$