1. Let $h = \{h_1, h_2, \ldots, h_k\}$ denote height in inches of k women. Also let $w = \{w_1, w_2, \ldots, w_k\}$ denote weight in pounds of these k women. Clearly h and w are correlated. Now let v_1 and v_2 denote eigenvectors of the covariance matrix of h and w. Define $SP_1 = Dv_1$ and $SP_2 = Dv_2$ where D is the matrix whose columns are h and w. Then correlation between SP_1 and SP_2 is

   (A) 0 (B) 1 (C) -1 (D) 0.5

2. Suppose the parametric form of a curve is given by $x = \dfrac{3u}{1+u^3}$ and $y = \dfrac{3u^2}{1+u^3}$. What is the implicit form of the same?

   (A) $x^2 + y^2 - 2xy = 0$
   (B) $x^3 + y^3 - 3xy = 0$
   (C) $x^3 + y^3 - 3x^2y = 0$
   (D) $x^3 + y^3 - 3x^2y - 3xy^2 = 0$

3. Consider a football match where all the 8 teams play between each other two times. Due to some pandemic, the tournament committee decided to reduce the number of matches. For this purpose, they divided the teams into two groups say Group A and Group B. Now each team has to play two matches between group members and one match each between other group members. In this case, how many matches can be saved altogether?

   (A) 12 (B) 14 (C) 16 (D) 32

4. Let A, B and C be events such that $P(A) = P(B) = 1/4$, $P(C) = 1/3$, $P(A \cap B) = 1/8$, $P(A \cap C) = 1/6$ and $P(B \cap C) = 0$. Then $P(A \cup B \cup C)$

   (A) is equal to 23/24
   (B) is equal to 17/24
   (C) is equal to 13/24
   (D) cannot be computed from the given information

5. Let $P = (1, 1)$, $Q = (3k, 2k + 1)$ and $R = (2k, k + 1)$ be three distinct points in the xy plane. If P, Q, R lie on the same line, then the value of k is

   (A) 1 (B) -1 (C) 2 (D) -2

6. The number of 5-digit integers formed by using the digits $0, 1, 3, 5, 7$ and 9 which are divisible by 10 and no digit is repeated is

   (A) 120 (B) 24 (C) 240 (D) 256

7. Let $X = \{2, 3, 4, \ldots, 21, 22, 23\}$. A number is chosen at random from the set X. The probability that it is a prime number more than 13 is

   (A) 9/22 (B) 3/22 (C) 1/3 (D) 1/9

8. Let a and b denote two vectors. Then the scalar and vector projections of a on b are

(A) $\frac{ab}{|b|}, \frac{ab}{|b|}\frac{b}{|b|}$

(B) $\frac{ab}{|b|}, \frac{ab}{|b|}\frac{a}{|a|}$

(C) $\frac{ab}{|a|}, \frac{ab}{|b|}\frac{b}{|b|}$

(D) $\frac{ab}{|a|}, \frac{ab}{|b|}\frac{a}{|a|}$

9. Let $p, q, r \in \mathbb{R}$. If $f(x) = px^2 + qx + r$ be such that $p + q + r = 3$ and $f(x+y) = f(x) + f(y) + xy$, for all $x, y \in \mathbb{R}$. Then the value of $f(5)$ is

(A) 25 (B) 30 (C) 35 (D) 40

10. To estimate proportion of a population which use a drug Marijuana, direct survey methods would not yield better estimate for the proportion because of the sensitiveness of the question. Let us say the following procedure is used to get a better estimate. As per this procedure, every participant of a sample set of participants of size n has to toss a fair coin. If the participant gets outcome of the toss as "head", he would answer the sensitive question (by saying either yes or no) otherwise he would answer an auxiliary question say whether his home door number is even. Let λ denote probability of a door number being even. If x denotes observed no of "yes" responses obtained from the tosses, then an estimate for proportion of the population who smokes the drug is

(A) $\frac{2x}{n} - \lambda$ (B) $\frac{2x}{n}$ (C) λ (D) $\frac{x}{n}$

11. Let A be a 3×3 non-singular matrix and $adj(A)$ denotes the adjoint of A. If $|2\ adj(A)| + 2|2A| + 8 = 0$, then $|A|$ is

(A) 1 (B) -1 (C) 2 (D) -2

12. Let the function $f(x)$ be defined as

$$f(x) = \begin{cases} ax + b & \text{if } x \leq -1, \\ 3x^2 + 2ax - b & \text{if } -1 < x \leq 1, \\ 4x & \text{if } x > 1. \end{cases}$$

Then $f(x)$ is continuous for all x, if

(A) $a = 1, b = 1$
(B) $a = 1, b = -1$
(C) $a = -1, b = 1$
(D) $a = -1, b = -1$

13. Let $u = (5, 4, 1)$, $v = (3, -4, 1)$ and $w = (1, -2, 3)$ be three vectors and consider the following statements.
(a) u and v are orthogonal
(b) v and w are orthogonal
(c) u and w are orthogonal

Which of the following are true?

(A) only (a) is correct
(B) only (b) is correct
(C) only (c) is correct
(D) both (a) and (c) are correct

14. Consider the following system of linear equations

$$\begin{aligned} \alpha^2 x - 2y + 3z &= -2 \\ \alpha x + 3y - z &= \beta \\ -2x - y - 2z &= \beta^2 \end{aligned}$$

Among the following the right choice for the pair (α, β) for which the above system has no solution is

(A) $(1, 1)$ (B) $(-2, 1)$ (C) $(-1, 2)$ (D) $(1, -1)$

15. Define $A_j = \sum_{i=1}^{n} i^j$, $j = 0, 1, 2, 3$. Then

$$\lim_{n\to\infty} \frac{A_1 A_2}{A_0 A_3}$$

is

(A) 0 (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) 1

16. A committee consisting of 5 distinct persons is to be selected from a group of 6 men and 8 women. The number of way the committee can be selected consisting of 2 men and 3 women is

(A) 560 (B) 840 (C) 48 (D) 6

17. Let $z = (1-t^2) + i\sqrt{1-t^2}$ be a complex number where t is a real number such that $|t| < 1$. Then the locus of z in the complex plane is

(A) an ellipse
(B) a hyperbola
(C) a parabola
(D) a pair of straight lines

18. The diameter of the circle $x^2 + y^2 - ax - by = 0$ is

(A) $\sqrt{a^2 + b^2}$ (B) $\sqrt{a^2 - b^2}$ (C) $a + b$ (D) $a - b$

19. Consider a 2×2 matrix of non-zero entries. It is known that the trace of the matrix is 10. If one of the eigenvalues is a prime number and the other is not (assuming the eigenvalues are positive), what is the value of the determinant of the matrix?

(A) 10 (B) 16 (C) 20 (D) 24

20. If α, β are roots of the quadratic equation $ax^2 + bx + c = 0$, then the roots of the quadratic $ax^2 + 3bx + 9c = 0$ are

(A) $3\alpha, 3\beta$ (B) $\alpha/3, \beta/3$ (C) $3\alpha, -3\beta$ (D) $\alpha/3, -\beta/3$

21. Let $f(x) = \dfrac{x}{x+2}$, $x \neq -2$. State which of the following statements is true.

(A) For all real y, there exists x such that $f(x) = y$

(B) For all real $y \neq -2$, there exists x such that $f(x) = y$

(C) For all real $y \neq 1$, there exists x such that $f(x) = y$

(D) None of the above is true

22. Suppose there are five cards in a box with labels $1, 2, 3, 4, 5$. First draw a card randomly from the box, note its label number and put it back in the box. Next draw a second card randomly from the box and note its label number. What is the probability that the absolute difference between these two numbers is odd?

(A) 1/2 (B) 11/25 (C) 12/25 (D) 13/25

23. Let C be the biggest circle lying in the first quadrant and touching X-axis, Y-axis and the line $4x - 3y + 6 = 0$. The diameter of the circle C is

(A) 3 (B) 3/2 (C) 4 (D) 2

24. If a focal chord of the parabola $x^2 = 4ay$ cuts it at two distinct points (x_1, y_1) and (x_2, y_2), then

(A) $x_1 x_2 = a^2$ (B) $y_1 y_2 = a^2$ (C) $y_1 y_2^2 = a^2$ (D) $y_1^2 y_2 = a^2$

25. Let A be a real matrix of order 8. If the rank of A is 5, then among the following the correct statement is

(A) The maximum possible rank of AA^T is 8
(B) The maximum possible rank of $A + A^T$ is 5
(C) The minimum possible rank of $A + A^T$ is 2
(D) The minimum possible rank of A^2 is 2

26. Consider two sets $A = \{2, 4, 6, 8, 10\}$ and $B = \{a, b, c\}$. How many different onto functions can be defined from A to B?

(A) 120 (B) 243 (C) 240 (D) 237

27. Let C be the circle through the foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. If the circle C passes through the point $(\sqrt{5}, 5)$ then the radius of C is

(A) $\frac{3\sqrt{5}}{2}$ (B) $3\sqrt{5}$ (C) $\frac{2\sqrt{5}}{3}$ (D) $\frac{5}{2}$

28. How many ways the letters in the word $ATTRIBUTE$ can be permuted so that all the three T's stay together?

(A) $\frac{9!}{3!}$ (B) $7!$ (C) $9!$ (D) $\frac{7!}{3!}$

29. Let $\frac{d}{dx}P(x) = \frac{e^{sinx}}{x}$, $x > 0$. If $\int_1^2 \frac{3}{x} e^{sinx^3} dx = P(k) - P(1)$, then which of the following is a possible value of k?

(A) 2 (B) 4 (C) 8 (D) 16

30. The area bounded by the curves $y = e^x$, $y = xe^x$ and the y-axis is

(A) $e - 2$ (B) $e + 2$ (C) $e - 1$ (D) $e - 3$