

POST-GRADUATE DIPLOMA IN STATISTICAL METHODS
AND ANALYTICS

TEST CODE: DST (Short Answer type) 2015

SYLLABUS

Algebra—Arithmetic and geometric progression, sequences, permutations and combinations, binomial theorem, theory of quadratic equations, inequalities, elementary set theory, vectors and matrices, determinant of a matrix, solutions of linear equations.

Coordinate geometry — Straight lines, circles.

Calculus— Functions of one variable, graphs of functions, differentiation and integration of functions, definite integrals, areas using integrals, maxima and minima.

SAMPLE QUESTIONS

1. Let A be a set of n elements. What is the number of ways we can choose an ordered pair (B, C) , where B, C are disjoint subsets of A ?
2. Let $(1+x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n$, n being a positive integer. Then find the value of

$$\left(1 + \frac{C_0}{C_1}\right) \left(1 + \frac{C_1}{C_2}\right) \dots \left(1 + \frac{C_{n-1}}{C_n}\right)$$

3. Find the number of positive integers which are less than or equal to 1000 and are divisible by none of 17, 19 and 23.
4. Write down the set $\{x : \left|x + \frac{1}{x}\right| > 6\}$ in the form of an interval.
5. If $f(x)$ is a real valued function such that

$$2f(x) + 3f(-x) = 15 - 4x,$$

for every $x \in \mathbb{R}$, then find the value of $f(2)$.

6. If $f(x) = \frac{\sqrt{3} \sin x}{2 + \cos x}$, then find the range of $f(x)$.
7. If M is a 3×3 matrix such that

$$[0 \ 1 \ 2]M = [1 \ 0 \ 0] \quad \text{and} \quad [3 \ 4 \ 5]M = [0 \ 1 \ 0]$$

then find $[6 \ 7 \ 8]M$.

8. Find the values of η for which the following system of equations

$$\begin{aligned}x + y + z &= 1 \\x + 2y + 4z &= \eta \\x + 4y + 10z &= \eta^2\end{aligned}$$

has a solution.

9. Find the number of divisors of 6000, where 1 and 6000 are also considered as divisors of 6000.
10. Let x_1 and x_2 be the roots of the quadratic equation $x^2 - 3x + a = 0$, and x_3 and x_4 be the roots of the quadratic equation $x^2 - 12x + b = 0$. If x_1, x_2, x_3 and x_4 ($0 < x_1 < x_2 < x_3 < x_4$) are in G.P., then find the value of ab .
11. Find:

$$\int_0^{\frac{\pi}{2}} \frac{\sin^{50} x}{\sin^{50} x + \cos^{50} x} dx$$

12. Find the value of ${}^n C_0 + 2 {}^n C_1 + 3 {}^n C_2 + \dots + (n+1) {}^n C_n$.
13. It is given that $e^a + e^b = 10$ where a and b are real. Then find the maximum value of $(e^a + e^b + e^{a+b} + 1)$.
14. Find the value of the following determinant $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix}$.
15. Determine the area enclosed by the curve $|x| + |y| = 1$.
16. For real α , find the value of the integral $\int_{\alpha}^{\alpha+1} [x] dx$, where $[x]$ denotes the largest integer less than or equal to x .
17. Consider 30 multiple-choice questions, each with four options of which exactly one is correct. What is the number of ways one can get only the alternate questions correctly answered?
18. Obtain the following sum of $n+1$ terms

$$2 + 3 \times \binom{n}{1} + 5 \times \binom{n}{2} + 9 \times \binom{n}{3} + 17 \times \binom{n}{4} + \dots \text{ up to } n+1 \text{ terms}$$

19. Find the value of the integral $I = 4 \times \left(\int_0^{\frac{\pi}{2}} \frac{e^{\cot^{-1}(\sin x)}}{e^{\cot^{-1}(\sin x)} + e^{\cot^{-1}(\cos x)}} dx \right)$.

20. Let A and B be disjoint sets containing m and n elements respectively, and let $C = A \cup B$. Then find the number of subsets S (of C) which contains p elements and also has the property that $S \cap A$ contains q elements.
21. Check whether the function $f(x) = x^{\frac{1}{x}}$, $x \neq 0$ has a maximum at $x = e$.
22. Find the number of permutations of the letters a, b, c and d such that b does not follow a , c does not follow b , and c does not follow d .
23. Find a local minima, if any, for $f(x) = \sin x^2$, $x \in \mathbb{R}$.
24. Use the properties such as monotonicity, convexity and maxima/minima (if any) of a function to draw a rough sketch of the graph of the function, $f(x)$ defined as $f(x) = x^3 - 6x^2 + 24x$, x being real.
25. Find the maximum value of the real valued function $f(x) = \cos x + \sin x$.
26. Calculate the value of the definite integral $\int_0^{\pi} \left| \frac{1}{2} + \cos x \right| dx$.
27. Find the set of real values of a such that the function $y = x^2 - ax + 1 - 2a^2$ takes positive values only for all real x .
28. Let $f(x) = \frac{x}{(x-1)(2x+3)}$, where, $x > 1$. Then find the 4th derivative of f .
29. Calculate the area under the curve $x^2 + 3x - 4$ in the positive quadrant and bounded by the line $x = 5$.
30. Find the number of real roots of the equation $1 + \cos^2 x + \cos^3 x - \cos^4 x = 5$.
31. If a, b are positive real variables whose sum is constant λ , then find the minimum value of $\sqrt{(1 + 1/a)(1 + 1/b)}$.
32. State the conditions on a, b and c under which the roots of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$, $b \neq 0$ and $c \neq 0$, are of unequal magnitude but of opposite signs.
- The following questions are of true/false type. Prove it if it is true or else give a counter-example to show that it is false.*
33. Check whether the following statement is true or false. Consider any integer $I = m^2 + n^2$, where m and n are odd integers. Then I is never divisible by 4.
34. Suppose that A is a 3×3 real matrix such that for each $u = (u_1, u_2, u_3)' \in \mathbb{R}^3$, $u' Au = 0$ where u' stands for the transpose of u . Then is the following true?

$$A' = A$$

35. Check if true or false: The function $f(x) = \sin x(1 + \cos x)$ which is defined for all real values of x has neither a maximum nor a minimum at $x = \pi/3$.
36. State whether true or false. The function $f(x)$ be defined as $f(x) = |x - 1| + |x - 2|$. $f(x)$ is differentiable at $x = 1$ but not at $x = 2$.