

Sample Question Paper-UGA

Group A

Each of the following questions have exactly one correct option and you have to identify it.

- The value of the integral $\int_{\pi/2}^{5\pi/2} \frac{e^{\tan^{-1}(\sin x)}}{e^{\tan^{-1}(\sin x)} + e^{\tan^{-1}(\cos x)}} dx$ equals
(a) 1. (B) π . (C) e . (D) none of these.
- The set of all solutions of the equation $\cos 2\theta = \sin \theta + \cos \theta$ is given by
(a) $\theta = 0$.
(b) $\theta = n\pi + \frac{\pi}{2}$, where n is any integer.
(c) $\theta = 2n\pi$ or $\theta = 2n\pi - \frac{\pi}{2}$ or $\theta = n\pi - \frac{\pi}{4}$, where n is any integer.
(d) $\theta = 2n\pi$ or $\theta = n\pi + \frac{\pi}{4}$, where n is any integer.
- For $k \geq 1$, the value of $\binom{n}{0} + \binom{n+1}{1} + \binom{n+2}{2} + \dots + \binom{n+k}{k}$ equals
(a) $\binom{n+k+1}{n+k}$. (B) $(n+k+1)\binom{n+k}{n+1}$. (C) $\binom{n+k+1}{n+1}$. (D) $\binom{n+k+1}{n}$.
- The value of $\sin^{-1} \cot \left[\sin^{-1} \left\{ \frac{1}{2} \left(1 - \sqrt{\frac{5}{6}} \right) \right\} + \cos^{-1} \sqrt{\frac{2}{3}} + \sec^{-1} \sqrt{\frac{8}{3}} \right]$ is
(a) 0. (B) $\pi/6$. (C) $\pi/4$. (D) $\pi/2$.
- If $a_n = \left(1 + \frac{1}{n^2}\right) \left(1 + \frac{2^2}{n^2}\right)^2 \left(1 + \frac{3^2}{n^2}\right)^3 \dots \left(1 + \frac{n^2}{n^2}\right)^n$, then
$$\lim_{n \rightarrow \infty} a_n^{-1/n^2}$$
is
(a) 0. (B) 1. (C) e . (D) $\sqrt{e}/2$.
- If $f(x) = e^x \sin x$, then $\left. \frac{d^{10}}{dx^{10}} f(x) \right|_{x=0}$ equals
(a) 1. (B) -1. (C) 10. (D) 32.
- Consider a circle with centre O . Two chords AB and CD extended intersect at a point P outside the circle. If $\angle AOC = 43^\circ$ and $\angle BPD = 18^\circ$, then the value of $\angle BOD$ is
(a) 36° . (B) 29° . (C) 7° . (D) 25° .
- A box contains 10 red cards numbered $1, \dots, 10$ and 10 black cards numbered $1, \dots, 10$. In how many ways can we choose 10 out of the 20 cards so that there are exactly 3 matches, where a match means a red card and a black card with the same number?
(a) $\binom{10}{3} \binom{7}{4} 2^4$. (B) $\binom{10}{3} \binom{7}{4}$. (C) $\binom{10}{3} 2^7$. (D) $\binom{10}{3} \binom{14}{4}$.
- Let P be a point on the ellipse $x^2 + 4y^2 = 4$ which does not lie on the axes. If the normal at the point P intersects the major and minor axes at C and D respectively, then the ratio $PC : PD$ equals
(a) 2. (B) $1/2$. (C) 4. (D) $1/4$.

10. The set of complex numbers z satisfying the equation

$$(3 + 7i)z + (10 - 2i)\bar{z} + 100 = 0$$

represents, in the Argand plane,

- (a) a straight line. (B) a pair of intersecting straight lines. (C) a pair of distinct parallel straight lines.
(D) a point.
11. The number of triplets (a, b, c) of integers such that $a < b < c$ and a, b, c are sides of a triangle with perimeter 21 is
(a) 7. (B) 8. (C) 11. (D) 12.
12. Suppose a, b and c are three numbers in G.P. If the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then $\frac{d}{a}, \frac{e}{b}$ and $\frac{f}{c}$ are in
(a) A.P. (B) G.P. (C) H.P. (D) none of the above.
13. The number of solutions of the equation $\sin^{-1} x = 2 \tan^{-1} x$ is
(a) 1. (B) 2. (C) 3. (D) 5.
14. Suppose $ABCD$ is a quadrilateral such that $\angle BAC = 50^\circ, \angle CAD = 60^\circ, \angle CBD = 30^\circ$ and $\angle BDC = 25^\circ$. If E is the point of intersection of AC and BD , then the value of $\angle AEB$ is
(a) 75° . (B) 85° . (C) 95° . (D) 110° .
15. Let \mathbb{R} be the set of all real numbers. The function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^3 - 3x^2 + 6x - 5$ is
(a) one-to-one, but not onto. (B) one-to-one and onto. (C) onto, but not one-to-one. (D) neither one-to-one nor onto.
16. Suppose $x, y \in (0, \pi/2)$ and $x \neq y$. Which of the following statements is true?
(a) $2 \sin(x + y) < \sin 2x + \sin 2y$ for all x, y .
(b) $2 \sin(x + y) > \sin 2x + \sin 2y$ for all x, y .
(c) There exist x, y such that $2 \sin(x + y) = \sin 2x + \sin 2y$.
(d) None of the above.
17. A triangle ABC has a fixed base BC . If $AB : AC = 1 : 2$, then the locus of the vertex A is
(a) a circle whose centre is the midpoint of BC .
(b) a circle whose centre is on the line BC but not the midpoint of BC .
(c) a straight line.
(d) none of the above.
18. Let N be a 50 digit number. All the digits except the 26th one from the right are 1. If N is divisible by 13, then the unknown digit is
(a) 1. (B) 3. (C) 7. (D) 9.
19. Suppose $a < b$. The maximum value of the integral
$$\int_a^b \left(\frac{3}{4} - x - x^2 \right) dx$$
over all possible values of a and b is
(a) $3/4$. (B) $4/3$. (C) $3/2$. (D) $2/3$.
20. For any $n \geq 5$, the value of $1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{2^n - 1}$ lies between
(a) 0 and $n/2$. (B) $n/2$ and n . (C) n and $2n$. (D) none of the above.

Group B

Each of the following questions may have multiple correct options and you have to identify all the correct options.

1. Let x be an irrational number. If a, b, c and d are rational numbers such that $\frac{ax+b}{cx+d}$ is a rational number, which of the following must be true?

- (A) $ad = bc$ (C) $ab = cd$.
 (B) $ac = bd$. (D) $a = d = 0$

2. Let $z = x + iy$ be a complex number, which satisfies the equation $(z + \bar{z})z = 2 + 4i$. Then

- (A) $y = \pm 2$. (C) $x = \pm 3$.
 (B) $x = \pm 2$. (D) $y = \pm 1$.

3. Define $S_n = \frac{1}{2} \cdot \frac{3}{4} \cdots \frac{2n-1}{2n}$ where n is a positive integer. Then

- (A) $S_n < \frac{1}{\sqrt{4n+2}}$ for some $n > 2$. (C) $S_n < \frac{1}{\sqrt{2n+5}}$ for all $n \geq 2$.
 (B) $S_n < \frac{1}{\sqrt{2n+1}}$ for all $n \geq 2$. (D) $S_n > \frac{1}{\sqrt{4n+2}}$ for all $n \geq 2$.

4. If $n^2 + 19n + 92$ is a perfect square, then the possible values of n may be

- (A) -19 (C) -4
 (B) -8 (D) -11

5. Let a, b and c be three real numbers. Then the equation $\frac{1}{x-a} + \frac{1}{x-b} + \frac{1}{x-c} = 0$

- (A) always have real roots. (C) always have real and equal roots.
 (B) can have real or complex roots depending on the values of a, b and c . (D) always have real roots, which are not necessarily equal.

6. Let X be the set $\{1, 2, 3, \dots, 10\}$ and P the subset $\{1, 2, 3, 4, 5\}$. The number of subsets Q of X such that $P \cap Q = \{3\}$ is

- (A) 1 (C) 2^5
 (B) 2^4 (D) 2^9

7. Suppose that the equations $x^2 + bx + ca = 0$ and $x^2 + cx + ab = 0$ have exactly one common non-zero root. Then

- (A) $a + b + c = 0$. (C) the two roots which are not common may not be real.
 (B) the two roots which are not common must necessarily be real. (D) the two roots which are not common are either both real or both not real.

8. Let $\frac{\tan(\alpha-\beta+\gamma)}{\tan(\alpha+\beta-\gamma)} = \frac{\tan\beta}{\tan\gamma}$. Then

- (A) $\sin(\beta - \gamma) = \sin(\alpha - \beta)$. (C) $\sin(\beta - \gamma) = 0$.
 (B) $\sin(\alpha - \gamma) = \sin(\beta - \gamma)$. (D) $\sin 2\alpha + \sin 2\beta + \sin 2\gamma = 0$

9. Let K be the set of all points (x, y) such that $|x| + |y| \leq 1$. Given a point A in the plane, let F_A be the point in K which is closest to A . Then the points A for which $F_A = (1, 0)$ are

- (A) all points $A = (x, y)$ with $x \geq 1$. (C) all points $A = (x, y)$ with $x \geq 1$ and $y = 0$.
 (B) all points $A = (x, y)$ with $x \geq y + 1$ and $x \geq 1 - y$. (D) all points $A = (x, y)$ with $x \geq 0$ and $y = 0$.

10. Let $\frac{\tan 3\theta}{\tan \theta} = k$. Then

- (A) $k \in (1/3, 3)$ (C) $\frac{\sin 3\theta}{\sin \theta} = \frac{2k}{k-1}$.
 (B) $k \notin (1/3, 3)$ (D) $\frac{\sin 3\theta}{\sin \theta} > \frac{2k}{k-1}$