

1. If $x = 1 + \sqrt[5]{2} + \sqrt[5]{4} + \sqrt[5]{8} + \sqrt[5]{16}$, then the value of $(1 + \frac{1}{x})^{30}$ is
- (A) 2 (B) 5 (C) 32 (D) 64
2. Let j be a number selected at random from $\{1, 2, \dots, 2024\}$. What is the probability that j is divisible by 9 and 15?
- (A) $\frac{1}{23}$ (B) $\frac{1}{46}$
(C) $\frac{1}{44}$ (D) $\frac{1}{253}$
3. Let S_n be the set of all n -digit numbers whose digits are all 1 or 2 and there are no consecutive 2's. (Example: 112 is in S_3 but 221 is not in S_3). Then the number of elements in S_{10} is
- (A) 512 (B) 256 (C) 144 (D) 89
4. There are 30 True or False questions in an examination. A student knows the answer to 20 questions and guesses the answers to the remaining 10 questions at random. What is the probability that the student gets exactly 24 answers correct?
- (A) $\frac{105}{2^9}$ (B) $\frac{105}{2^8}$ (C) $\frac{105}{2^{10}}$ (D) $\frac{4}{2^{10}}$
5. Let T be a right-angled triangle in the plane whose side lengths are in a geometric progression. Let $n(T)$ denote the number of sides of T that have integer lengths. Then the maximum value of $n(T)$ over all such T is
- (A) 0 (B) 1 (C) 2 (D) 3

6. Let x_1, x_2, \dots, x_n be non-negative real numbers such that $\sum_{i=1}^n x_i = 1$. What is the maximum possible value of $\sum_{i=1}^n \sqrt{x_i}$?

- (A) 1 (B) \sqrt{n} (C) $n^{3/4}$ (D) n

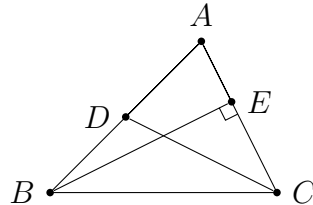
7. The precise interval on which the function $f(x) = \log_{1/2}(x^2 - 2x - 3)$ is monotonically decreasing, is

- (A) $(-\infty, -1)$ (B) $(-\infty, 1)$
 (C) $(1, \infty)$ (D) $(3, \infty)$

8. The angle subtended at the origin by the common chord of the circles $x^2 + y^2 - 6x - 6y = 0$ and $x^2 + y^2 = 36$ is

- (A) $\pi/2$ (B) $\pi/4$ (C) $\pi/3$ (D) $2\pi/3$

9. In $\triangle ABC$, CD is the median and BE is the altitude. Given that $\overline{CD} = \overline{BE}$, what is the value of $\angle ACD$?



- (A) $\pi/3$ (B) $\pi/4$ (C) $\pi/5$ (D) $\pi/6$

10. If the points z_1 and z_2 are on the circles $|z| = 2$ and $|z| = 3$, respectively, and the angle included between these vectors is 60° , then the value of $\frac{|z_1 + z_2|}{|z_1 - z_2|}$ is

- (A) $\sqrt{\frac{19}{7}}$ (B) $\sqrt{19}$ (C) $\sqrt{7}$ (D) $\sqrt{\frac{7}{19}}$

11. Let $n \geq 1$. The maximum possible number of primes in the set $\{n + 6, n + 7, \dots, n + 34, n + 35\}$ is

- (A) 7 (B) 8 (C) 12 (D) 13

12. Suppose 40 distinguishable balls are to be distributed into 4 different boxes such that each box gets exactly 10 balls. Out of these 40 balls, 10 are defective and 30 are non-defective. In how many ways can the balls be distributed such that all the defective balls go to the first two boxes?

- (A) $\frac{40!}{(10!)^4}$ (B) $\frac{30! \cdot 20!}{(10!)^5}$ (C) $\frac{20! \cdot 20!}{(10!)^5}$ (D) $\frac{30! \cdot 10!}{(10!)^4}$

13. The number of elements in the set

$$\{x : 0 \leq x \leq 2, |x - x^5| = |x^5 - x^6|\}$$

is

- (A) 2 (B) 3 (C) 4 (D) 5

14. In a room with $n \geq 2$ people, each pair shakes hands between themselves with probability $\frac{2}{n^2}$ and independently of other pairs. If p_n is the probability that the total number of handshakes is at most 1, then $\lim_{n \rightarrow \infty} p_n$ is equal to

- (A) 0 (B) 1 (C) e^{-1} (D) $2e^{-1}$

15. The number of positive solutions to the equation

$$e^x \sin x = \log x + e^{\sqrt{x}} + 2$$

is

- (A) 0 (B) 1 (C) 2 (D) ∞

16. Let $n > 1$ be the smallest composite integer that is coprime to $\frac{10000!}{9900!}$. Then

- (A) $n \leq 100$ (B) $100 < n \leq 9900$
(C) $9900 < n \leq 10000$ (D) $n > 10000$

17. Let $P = \{(x, y) : x + 1 \geq y, x \geq -1, y \geq 2x\}$. Then the minimum value of $(x + y)$ where (x, y) varies over the set P is

- (A) -1 (B) -3 (C) 3 (D) 0

18. Let $A = \{1, \dots, 5\}$ and $B = \{1, \dots, 10\}$. Then the number of ordered pairs (f, g) of functions $f : A \rightarrow B$ and $g : B \rightarrow A$ satisfying $(g \circ f)(a) = a$ for all $a \in A$ is

- (A) $\frac{10!}{5!} \times 5^5$ (B) $5^{10} \times 5!$ (C) $10! \times 5!$ (D) $\binom{10}{5} \times 10^5$

19. Let

$$S = \frac{1}{\sqrt{10000}} + \frac{1}{\sqrt{10001}} + \dots + \frac{1}{\sqrt{160000}}.$$

Then the largest positive integer not exceeding S is

- (A) 200 (B) 400 (C) 600 (D) 800

20. The real number x satisfies

$$\frac{|x|^2 - |x| - 2}{2|x| - |x|^2 - 2} > 2$$

if and only if x belongs to

- (A) $(-2, -1) \cup (1, 2)$ (B) $(-2/3, 0) \cup (0, 2/3)$
(C) $(-1, -2/3) \cup (2/3, 1)$ (D) $(-1, 0) \cup (0, 1)$

21. Consider points of the form (n, n^k) , where n and k are integers with $n \geq 0, k \geq 1$. How many such points are strictly inside the circle of radius 10 with centre at the origin?
- (A) 11 (B) 12 (C) 15 (D) 17
22. Let $n > 1$, and let us arrange the expansion of $(x^{1/2} + \frac{1}{2x^{1/4}})^n$ in decreasing powers of x . Suppose the first three coefficients are in arithmetic progression. Then, the number of terms where x appears with an integer power, is
- (A) 3 (B) 2 (C) 1 (D) 0
23. The limit
- $$\lim_{n \rightarrow \infty} \frac{2 \log 2 + 3 \log 3 + \cdots + n \log n}{n^2 \log n}$$
- is equal to
- (A) 0 (B) 1/4 (C) 1/2 (D) 1
24. Let $p < q$ be prime numbers such that $p^2 + q^2 + 7pq$ is a perfect square. Then, the largest possible value of q is:
- (A) 7 (B) 11 (C) 23 (D) 29
25. The set of all real numbers x for which $3^{2^{1-x^2}}$ is an integer has
- (A) 3 elements (B) 15 elements
(C) 24 elements (D) infinitely many elements

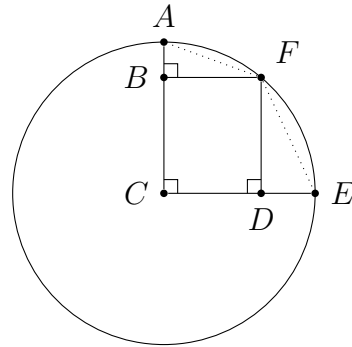
26. Let a, b, c be three complex numbers. The equation

$$az + b\bar{z} + c = 0$$

represents a straight line on the complex plane if and only if

- (A) $a = b$ (B) $\bar{a}c = b\bar{c}$
 (C) $|a| = |b| \neq 0$ (D) $|a| = |b| \neq 0$ and $\bar{a}c = b\bar{c}$

27. In the adjoining figure, C is the centre of the circle drawn, A, F, E lie on the circle and $BCDF$ is a rectangle. If $\frac{DE}{AB} = 2$, then $\frac{FE}{FA}$ equals



- (A) $\sqrt{\frac{3}{2}}$ (B) $\sqrt{2}$
 (C) $\sqrt{\frac{5}{2}}$ (D) $\sqrt{3}$

28. For every increasing function $b : [1, \infty) \rightarrow [1, \infty)$ such that

$$\int_1^\infty \frac{dx}{b(x)} < \infty,$$

we must have

- (A) $\sum_{k=1}^\infty \frac{\sqrt{\log k}}{b(k)} < \infty$ (B) $\sum_{k=3}^\infty \frac{\log k}{b(\log k)} < \infty$
 (C) $\sum_{k=1}^\infty \frac{e^k}{b(e^k)} < \infty$ (D) $\sum_{k=3}^\infty \frac{1}{\sqrt{b(\log k)}} < \infty$

29. Consider the following two statements:

- (I) There exists a differentiable function $g : \mathbb{R} \rightarrow \mathbb{R}$ such that $g(x^3 + x^5) = e^x - 100$.
- (II) There exists a continuous function $g : \mathbb{R} \rightarrow \mathbb{R}$ such that $g(e^x) = x^3 + x^5$.

Then

- (A) Only (I) is correct.
- (B) Only (II) is correct.
- (C) Both (I) and (II) are correct.
- (D) Neither (I) nor (II) is correct.

30. Let $\psi : \mathbb{R} \rightarrow \mathbb{R}$ be a continuous function with $\int_{-1}^1 \psi(x) dx = 1$.
Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function. Then

$$\lim_{\varepsilon \rightarrow 0} \frac{1}{\varepsilon} \int_{1-\varepsilon}^{1+\varepsilon} f(y) \psi\left(\frac{1-y}{\varepsilon}\right) dy$$

equals

- (A) $f(1)$
- (B) $f(1)\psi(0)$
- (C) $f'(1)\psi(0)$
- (D) $f(1)\psi(1)$